Temporal dynamics of alcohol consumption patterns: The peer pressure and binge drinkers' role

Dinámica temporal de los patrones de consumo de alcohol: presión de pares y el rol de bebedores compulsivos

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Abstract—Alcohol consumption is a problem of both social and health interest since consumption at an early age increases the probability of developing alcohol dependence, along with a series of risks associated with diseases, violence, and injuries. In young people, the first episode and recurrence of alcohol consumption usually occur in the form of binge drinking, in which peers assume a protective or risky role. Mathematical modeling of binge drinking has frequently been performed based on interactions with other consumption patterns, defined in terms of quantity and frequency, without considering that the periodicity of excessive (or compulsive) alcohol consumption is associated with specific social contexts, such as parties, where mainly social drinkers adopt this pattern. Our objective is to analyze the influence exerted by social drinkers on their peers who adopt excessive alcohol consumption, as well as the recurrence and persistence of harmful consumption. We formulate a mathematical model described by a Filippov system, where the "contagion" dynamic is based on two transfer sequences according to the workweek and weekend. Our findings establish that depending on the parameter values of the model, four asymptotic periodic dynamics can arise. In addition to this, the existence of a trade-off between protective and risk factors is evidenced, allowing evaluation of the effect of social variables on binge drinking prevalence.

Keywords—Alcohol abuse, Excessive alcohol consumption, Filippov system, Trade-off

Resumen—El consumo de alcohol es un problema de interés tanto social como sanitario, ya que un consumo a edades tempranas aumenta la probabilidad de desarrollar dependencia al alcohol, junto con una serie de riesgos asociados a enfermedades, violencia y lesiones. En los jóvenes, el primer episodio y la recurrencia del consumo de alcohol suelen darse en forma de borracheras, en las que los pares asumen un rol protector o de riesgo. La modelización matemática del consumo excesivo de alcohol ha sido realizada habitualmente a partir de interacciones con otros patrones de consumo, definidos en función de cantidad y frecuencia, sin considerar que la periodicidad del consumo excesivo (o compulsivo) está asociada a contextos sociales específicos, tales como fiestas, donde principalmente bebedores sociales adoptan este patrón. Nuestro objetivo es analizar la influencia que ejercen bebedores sociales en sus pares, quienes adoptan un consumo excesivo de alcohol, además de la recurrencia y la persistencia hacia un consumo nocivo. Formulamos un modelo matemático descrito por un sistema de Filippov, donde la dinámica de "contagio" se basa en dos secuencias de transferencia en correspondencia a la semana laboral y el fin de semana. Nuestros hallazgos establecen que, dependiendo de los valores de los parámetros del modelo, pueden surgir cuatro dinámicas periódicas asintóticas. Además de eso, se evidencia la existencia de un trade-off entre los factores protectores y de riesgo, lo que permite evaluar el efecto de las variables sociales en la prevalencia del consumo excesivo de alcohol.

Palabras clave— Abuso de alcohol, Consumo excesivo de alcohol, Sistema de Filippov, Compromiso

INTRODUCTION

Alcohol is a drug highly consumed worldwide due to legal character and regulatory frameworks based on permissive laws and policies (Brown et al., 2008; Margozzini and Sapag, 2015). Harmful alcohol consumption is responsible for multiple diseases and injuries (World Health Organization, 2019), whose consequences affect the person who consumes it and their immediate environments, such as family, friends, co-workers, and neighbors (Bernstein et al., 2007; Sophie, 2019).

Based on a drink standard unit established by World Health Organization, equivalently to 10 grams of pure alcohol (World Health Organization, 2019), a person who consumes alcohol can be categorized as a Social drinker (also termed moderate drinker), Risk drinker (also termed frequent drinker), Harmful drinker, or Alcoholic drinker (also termed, Dependent drinker) according to the quantity, frequency, and duration of alcohol consumption (Benedict, 2007; Sánchez et al., 2007; Brauer, 2008). The change in the consumption pattern from one individual to another of a higher hierarchy, ascending both in quantity and frequency of alcohol consumption, has followed the medical/health model where a drug user is a sick person who must be cured. In this approach, the drug is an infectious-contagious agent that acts on the individual (Leiva-Vásquez and Rojas-Jara, 2018) and, therefore, potentially transmissible to other individuals, often called susceptible, which in this framework corresponds to social drink and abstainers.

The mathematical modeling used in studying temporal dynamics of alcohol consumption patterns has followed principles and guidelines of the mathematical modeling approach in infectious and contagious diseases (Benedict, 2007), based on ordinary differential equations (An der Heiden et al., 1998; Sánchez et al., 2007; Benedict, 2007; Manthey et al., 2008; Santonja et al., 2010; Sharma and Samanta, 2013; Walters et al., 2013; Bani et al., 2013; Buonomo and Lacitignola, 2014; Sharma and Samanta, 2015; Adu et al., 2017; Giacobbe et al., 2017; An et al., 2020; Crokidakis and Sigaud, 2021; Bentout et al., 2021) and difference equations (Khajji et al., 2020d,c; Labzai et al., 2020; El Youssoufi et al., 2021; Gutiérrez et al., 2022). In addition, these models also incorporate delay processes (Ma et al., 2015; Hai-Feng et al., 2017; Buonomo et al., 2018; Zhang et al., 2020; Ma et al., 2021; Djillali et al., 2021), stochastic processes (Wang et al., 2017; Anwarud and Yongjin, 2021), impulsive processes (Scribner et al., 2009; Lakshmikantham et al., 1989), or optimal processes (Bonyah et al., 2019; Khajji et al., 2020a; Pérez, 2020; El Youssoufi et al., 2021). Traditionally, the interconnection among consumption patterns follows a transference sequence: Social drinkers $\hookrightarrow$ Risk drinkers $\hookrightarrow$ Harmful drinkers (or Binge drinkers) $\hookrightarrow$ Alcoholic drinkers, with the possibility of returning to previous categories or being removed depending on the successful application of detoxification practices (Khajji et al., 2020c,b). In addition, the co-abuse of substances (*e.g.*, alcohol and smoking or alcohol and methamphetamine) (Bhunu and Mushayabasa, 2012; Orwa and Nyabadza, 2019) and the role of alcohol consumption in the transmission and progression of various diseases (Thomas and Lungu, 2009; Bowong et al., 2011; Mushayabasa and Bhunu, 2011; Bonyah et al., 2019) have been analyzed from mathematical models that incorporate the drinking patterns in its formulation.

Several studies on alcohol consumption reveal concern about excessive alcohol consumption (Parada et al., 2011; Llerena et al., 2015), characterized by consumption of 60/50 grams of alcohol pure per man/woman, carried out episodically and concentrated in a short period (two hours) by adolescent groups without gender differentiation on weekend nights (Parada et al., 2011). Furthermore, socio-medical investigations closely link this consumption pattern with social drinker individuals (Reifman et al., 1998; Adan et al., 2017; Llerena et al., 2015), stand out the role of peers in the first drunk episode and behavioral persistence (Borsari and Carey, 2001; Christiansen et al., 2002; Duncan et al., 2005; Rojas-Jara and Leiva-Vásquez, 2018; González-Araya and Rojas-Jara, 2020). Thus, the binge drinker label can be understood as a momentary behavior that social drinkers mainly adopt in specific drink circumstances such as parties and familiar or friend meetings (Sudhinaraset et al., 2016; Bahr et al., 1995). Importantly, the consequences of excessive alcohol consumption on health vary, highlighting long-term memory loss and muscle problems in the long-term (Crews et al., 2016; Hermens and Lagopoulos, 2018; Föger-Samwald et al., 2018; Voskoboinik et al., 2021; Degerud et al., 2021).

In the mathematical modeling framework, and based on the quantity consumption criterion for defining the compartments into a supposed alcoholic population, the Binge drinker and Harmful drinker classes are equivalents. The Binge drinking pattern has been studied from interconnections with the other compartments, particularly influencing social drinkers (Mulone and Straughan, 2012; Huo and Song, 2012; Anwarud and Yongjin, 2021) or being influenced by Risk drinkers (Hai-Feng et al., 2017; Zhang et al., 2020; Anwarud and Yongjin, 2021). However, when considering both the frequency and duration of consumption in specific circumstances, binge drinking behavior emerges mainly from Social drinkers (Parada et al., 2011). The above approach is conceptually correct and has not been widely explored. On this matter, in Gutiérrez et al. (2022) using a discrete-time mathematical model, analyzing the proportion of social drinkers who engage in excessive drinking behavior in social contexts based on parameters associated with social vulnerability (Rimal and Real, 2005; Margozzini and Sapag, 2015). Their findings, apart from the analysis and dynamic richness of the proposed model, establish parametric relationships that guarantee the predominance of social consumption, where peer pressure plays an important role. Outstanding research has considered peer pressure, inserting it into the point prevalence as a function dependent on external variable information that favors or discourages binge drinking (Giacobbe et al., 2017; Buonomo et al., 2018) and whose mechanism is inspired by imitation of inappropriate behavior and social conformity (Buonomo and Lacitignola, 2014; Straughan, 2019).

We formulate a switch and impulsive mathematical model (a Filippov system) to represent the alcohol consumption "contagion" dynamics based on two transference sequences according to the work week and weekend interactions:

(1) Social drinkers $\hookrightarrow$ Risk drinkers $\hookrightarrow$ Harmful drinkers,

(2) Social drinkers $\hookrightarrow$ Binge drinkers and Risk drinkers $\hookrightarrow$ Harmful drinkers,

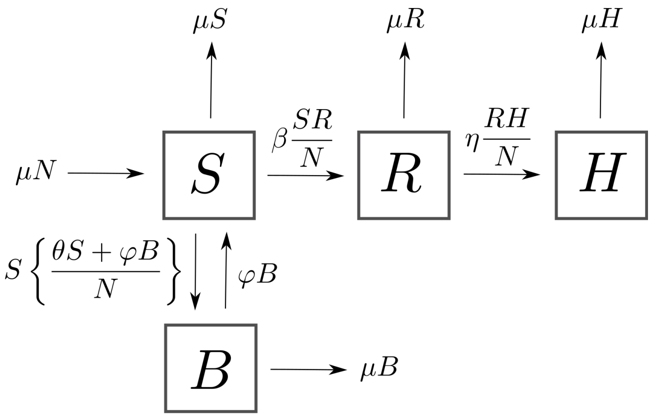


Figure 1: Progression diagram for alcoholic drinking compartmental mathematical model where S, R, H, and B correspond to Social drinkers, Risk drinkers, Harmful drinkers, and Binge drinkers, respectively.

Our goal is to analyze (i) the negative peer influence induced by social drinkers on other social drinkers for binge drinking adopt, (ii) the effect of binge drinkers on social drinkers, (iii) the recurrence of binge drinking, and (iv) the persistence on harmful consumption from a binge consumption on the temporal dynamics of alcohol.

The organization of this paper is as follows. In Section 2, a preliminary alcohol consumption mathematical model based on ordinary differential equations is formulated, and its qualitative analysis is carried out, from which a switch and impulsive model emerges as a consequence of differentiating the interrelations among drinking classes based on periodicity and contexts that favor the binge drinking by social drinkers. Then, in Section 3, a parametric sensibility analysis of temporal dynamics of Social and Binge drinking patterns is carried out. Finally, our findings are discussed in Section 4 as possibilities for future work.

Mathematical model formulation

A preliminary model and main results

Let be $S(t)$, $R(t)$, $H(t)$, and $B(t)$ compartments of Social drinkers, Risk drinkers, Harmful drinkers, and Binge drinkers at time t, respectively. We assumed a constant population size denoted by N, such that $S(t)+R(t)+H(t)+B(t)=N$ is obtained. New drinkers adopt social drinking at a rate proportional to N, with a constant given by μ, where μ is a unique vital rate used to represent both inflow and outflow of individuals in the compartments. From the traditional mathematical approach used for modeling the infectious and contagious disease dynamics (Brauer, 2017), alcohol drinking patterns are formulated (Sánchez et al., 2007). Thus, a social drinker individual becomes a risk drinker. In turn, a risk drinker individual becomes a harmful drinker by the negative influence, defined by effective contact between individuals of different compartments at rates $\beta S(R/N)$ and $\eta R(H/N)$ respectively, that promote the change of alcohol consumption pattern. In turn, social drinkers adopt a binge drinking pattern at a rate $S\{(\theta S+\varphi B)/N\}$ where θ is the peer pressure rate, and φB is the rate of returning to social consumption. Figure 1 illustrates the interconnections among these drinkers classes.

Therefore, the following model is proposed

$$\begin{cases} S'(t) = \mu N - \beta S(t)\dfrac{R(t)}{N} - S(t)\left\{\dfrac{\theta S(t)+\varphi B(t)}{N}\right\} + \\ \qquad +\varphi B(t) - \mu S(t) \\ R'(t) = +\beta S(t)\dfrac{R(t)}{N} - \eta R(t)\dfrac{H(t)}{N} - \mu R(t) \\ H'(t) = +\eta R(t)\dfrac{H(t)}{N} - \mu H(t) \\ B'(t) = +S(t)\left\{\dfrac{\theta S(t)+\varphi B(t)}{N}\right\} - \varphi B(t) - \mu B(t) \end{cases}, \tag{1}$$

with $S(0)+R(0)+H(0)=N$ and $B(0)=0$.

We observed that not exist alcohol-free equilibrium point due to $\theta \neq 0$. Even more, the equilibrium points of the normalized model (1) are:

(1) $E_1=(S_1^*,0,0,1-S_1^*)$ where

$$S_1^* = \frac{2(\mu+\varphi)}{\mu+2\varphi+\sqrt{\mu^2+4\theta(\mu+\varphi)}}.$$

(2) $E_2=(S_2^*,1-S_2^*-B_2^*,0,B_2^*)$ where

$$S_2^*=\frac{\mu}{\beta}, \quad \text{and} \quad B_2^*=\frac{\theta\mu^2}{\beta[\beta(\mu+\varphi)-\mu\varphi]},$$

with $\beta/\mu>1$.

(3) $E_3=(S_3^*,R_3^*,H_3^*,B_3^*)$ where

$$S_3^* = \frac{2\eta(\mu+\varphi)}{(\beta+\eta)(\mu+\varphi)+\varphi\eta+\sqrt{[\beta(\mu+\varphi)+\mu\eta]^2+4\theta\eta^2(\mu+\varphi)}},$$

$$R_3^* = \frac{\mu}{\eta}, \quad H_3^*=\frac{\beta S_3^*-\mu}{\eta}, \quad B_3^*=\frac{\theta {S_3^*}^2}{\mu+\varphi(1-S_3^*)},$$

with $\eta/\mu>1$. In addition, note that $0<S_3^*<1$. Indeed, taking $\xi=\beta/\eta$, then

$$S_3^*=\frac{2(\mu+\varphi)}{\xi(\mu+\varphi)+\mu+2\varphi+\sqrt{[\xi(\mu+\varphi)+\mu]^2+4\theta(\mu+\varphi)}}.$$

Defining S_3^* as a function of $\xi>0$, we have that $dS_3^*/d\xi<0$. Thus,

$$S_3^* < \frac{2(\mu+\varphi)}{\mu+2\varphi+\sqrt{\mu^2+4\theta(\mu+\varphi)}} = S_1^* < 1$$

is obtained, such that $S_3^*\to 0$ as ξ increases.

Let be threshold values

$$\mathscr{U} = \frac{2\beta(\mu+\varphi)}{\mu\{\mu+2\varphi+\sqrt{\mu^2+4\theta(\mu+\varphi)}\}},$$

$$\mathscr{V} = \frac{\eta}{\mu}\left\{1-\frac{\mu}{\beta}-\frac{\theta\mu^2}{\beta[\beta(\mu+\varphi)-\mu\varphi]}\right\}.$$

Then, the following result establishes the stability of each equilibrium point.

Proposition 1 *Consider model (1). Thus,*

(1) If $0<\mathscr{U}<1$, then the equilibrium point E_1 is locally asymptotically stable.

(2) If $\mathscr{U} > 1$ and $0 < \mathscr{V} < 1$ then the equilibrium point E_2 is locally asymptotically stable.

(3) If $\mathscr{U} > 1$ and $\mathscr{V} > 1$ then the equilibrium point E_3 is locally asymptotically stable.

Proof 1 *Let be $J(E)$ the Jacobian matrix associate with model (1) at equilibrium point $E = (S, R, H, B)$.*

(1) The eigenvalues of $J(E_1)$ are

$\left\{-\mu, -\mu, \lambda_0, -\sqrt{\mu^2+4\theta(\mu+\varphi)}\right\}$ *where*

$$\lambda_0 = \frac{2\varphi(\beta-\mu)^2 - 2\mu[(\beta+\theta)\mu - \beta^2]}{\beta(\mu+2\varphi)+2\mu(\theta-\varphi)+\beta\sqrt{\mu^2+4\theta(\mu+\varphi)}}.$$

Taking $\beta = \xi\mu$, $\theta = \tau\mu$ and $\varphi = \phi\mu$ we have

$$\begin{aligned}\lambda_0 &= \mu\frac{-[\xi+2\tau+2\phi(\xi-1)]+\xi\sqrt{1+4\tau(1+\phi)}}{2(\tau-\phi)},\\ &= -\mu\frac{2\{\tau+(1-\xi)[\xi(1+\phi)-\phi]\}}{2(\tau-\phi)+\xi(1+2\phi)+\xi\sqrt{1+4\tau(1+\phi)}}.\end{aligned}$$

From $0 < \mathscr{U} < 1$, follows

$$\xi < \frac{1}{2}\left\{1+\frac{\phi+\sqrt{1+4\tau(1+\phi)}}{1+\phi}\right\}.$$

Let be $\rho > 0$ such that

$$\xi = \frac{1}{2}\left\{1+\frac{\phi+\sqrt{1+4\tau(1+\phi)}}{1+\phi}\right\} - \rho.$$

Then,

$$\lambda_0 = -\mu\rho\frac{2(1+\phi)}{1+2\varphi+\sqrt{1+4\tau(1+\phi)}} < 0.$$

Therefore, E_1 is locally asymptotically stable.

(2) The eigenvalues of $J(E_2)$ are

$$\left\{-\mu, \lambda_1, \lambda_2, -\mu\underbrace{\left[1-\frac{\eta}{\mu}\left(1-\frac{\mu}{\beta}-\frac{\theta\mu^2}{\beta[\beta(\mu+\varphi)-\mu\varphi]}\right)\right]}_{1-\mathscr{V}}\right\},$$

where λ_1 and λ_2 are roots of $P(\lambda) = \lambda^2 + p_1\lambda + p_2$ with

$$\begin{aligned}p_1 &= \frac{\beta^2\varphi^2-\mu^2\varphi(\theta-\varphi)+\beta^3(\mu+\varphi)+\beta\mu(\theta-\varphi)(\mu+2\varphi)}{\beta[\beta(\mu+\varphi)-\mu\varphi]},\\ p_0 &= \frac{\beta^2(\mu+\varphi)-\mu[\mu(\tau-\varphi)+\beta(\mu+2\varphi)]}{\beta}.\end{aligned}$$

Taking $\beta = \xi\mu$ (Here, $\xi > 1$), $\theta = \tau\mu$, $\varphi = \phi\mu$ and from $\mathscr{U} > 1$, we have

$$\xi > \frac{1}{2}\left\{1+\frac{\phi+\sqrt{1+4\tau(1+\phi)}}{1+\phi}\right\}.$$

Let be $\rho > 0$ such that

$$\xi = \frac{1}{2}\left\{1+\frac{\phi+\sqrt{1+4\tau(1+\phi)}}{1+\phi}\right\} + \rho,$$

then $p_1 = 2\mu\{\rho^2(1+\phi)[2\rho(1+\phi)+3(1+\sqrt{1+4\tau(1+\phi)})+2\phi(3+\phi)]+(1+\phi)+(1+\phi)[\sqrt{1+4\tau(1+\phi)}+2\tau(2+2\phi+\sqrt{1+4\tau(1+\phi)})]+4\rho\tau(1+\phi)(2+\phi)+\rho[3(1+\sqrt{1+4\tau(1+\phi)})+2\phi(2+\phi+3\sqrt{1+4\tau(1+\phi)}+\phi\sqrt{1+4\tau(1+\phi)})]\}/\{(1+2\rho(1+\phi)+\sqrt{1+4\tau(1+\phi)})(1+2\phi+2\rho(1+\phi)+\sqrt{1+4\tau(1+\phi)})\}$ and $p_0 = 2\rho\mu^2(1+\phi)[\rho(1+\phi)+\sqrt{1+4\tau(1+\phi)}]/\{1+2\phi+2\rho(1+\phi)+\sqrt{1+4\tau(1+\phi)}\}$, both positive values. Then, by the Routh–Hurwitz criterion, E_2 is locally asymptotically stable.

(3) The eigenvalues of $J(E_3)$ are $\{-\mu, \lambda_3, \lambda_4, \lambda_5\}$, where

λ_3, λ_4 and λ_5 are roots of $Q(\lambda) = \lambda^3 + q_2\lambda^2 + q_1\lambda + q_0$ with

$$\begin{aligned}q_2 &= \frac{r(S_3^*)}{\mu+\varphi(1-S_3^*)}+\frac{\beta\mu}{\eta},\\ q_1 &= \frac{\mu\{(\beta S_3^*-\mu)\eta+\beta[\beta S_3^*+\varphi(1-S_3^*)]\}}{\eta},\\ q_0 &= \frac{\mu(\beta S_3^*-\mu)\{\beta[\mu+\varphi(1-S_3^*)]^2+\eta r(S_3^*)\}}{\eta[\mu+\varphi(1-S_3^*)]},\end{aligned}$$

and $r(S) = -\varphi(\theta-\varphi)S^2 + 2(\theta-\varphi)(\mu+\varphi)S + (\mu+\varphi)^2$ which satisfies

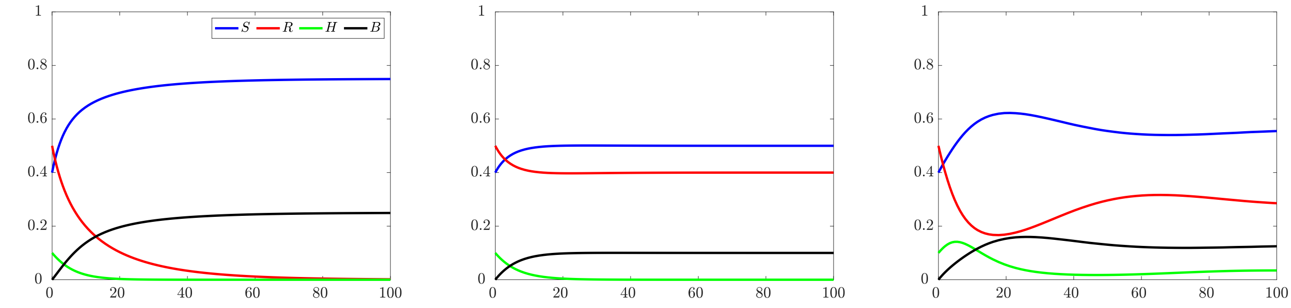
(i) If $\theta = \varphi$, then $r(S) = (\mu+\varphi)^2 > 0$,

(ii) If $\theta > \varphi$, then $r(S) > \text{mín}\{r(0), r(1)\}$ with $r(0) = (\mu+\varphi)^2 > 0$ and $r(1) = \mu^2 + \theta(2\mu+\varphi) > 0$.

(iii) If $\theta < \varphi$ then $r(S) \geq (3\theta+\varphi)(\mu+\varphi)^2/(4\varphi) > 0$ for any $S > 0$.

Consequently, $q_2 > 0$. On the other hand, $q_1 > 0$ and $q_0 > 0$ if and only if $\beta S_3^* - \mu > 0$ (i.e., the existence condition of equilibrium point E_3, positivity of component B_3^*). Indeed,

$$\begin{aligned}\beta S_3^* - \mu &= \frac{\beta[\beta(\mu+\varphi)+(\mu+2\varphi)\eta]-2\mu[(\beta+\eta)\varphi-\theta\eta)]-\beta\sqrt{\beta^2(\mu+\varphi)^2+2\beta\eta\mu(\mu+\varphi)+[\mu^2+4\theta(\mu+\varphi)]\eta^2}}{2[(\beta+\eta)\varphi-\theta\eta)]},\\ &= \frac{2\{\beta^2(\mu+\varphi)(\eta-\mu)+\mu^2(\varphi-\theta)\eta-\beta\mu[2\varphi\eta+\mu(\eta-\varphi)]\}}{\underbrace{\beta[\beta(\mu+\varphi)+(\mu+2\varphi)\eta]-2\mu[(\beta+\eta)\varphi-\theta\eta)]}_{N}+\underbrace{\beta\sqrt{\beta^2(\mu+\varphi)^2+2\beta\eta\mu(\mu+\varphi)+[\mu^2+4\theta(\mu+\varphi)]\eta^2}}_{L}} = \frac{M}{N+L},\end{aligned}$$



(a) (b) (c)

Figure 2: Temporal dynamics of model (1) in correspondence to outcomes in Proposition 1. We assumed $N = 1$ and initial conditions $S(0) = 0.4$, $R(0) = 0.5$, $H(0) = 0.1$, and $B(0) = 0$, with common parameters: $\mu = 0.2$, $\varphi = 0.1$, and $\theta = 0.1$. In particularly, (a) $\beta = 0.2$ and $\eta = 0.1$ with $\mathscr{U} = 0.75$, (b) $\beta = 0.4$ and $\eta = 0.1$ with $\mathscr{U} = 1.5$ and $\mathscr{V} = 0.2$, and (c) $\beta = 0.4$ and $\eta = 0.7$ with $\mathscr{U} = 1.5$ and $\mathscr{V} = 1.4$.

where clearly $L > 0$. Let be $\beta = \xi\mu$ (Here, $\xi > 1$), $\theta = \tau\mu$, $\varphi = \phi\mu$ and $\eta = \kappa\mu$. Because $\mathscr{V} > 1$, exist $\sigma > 1$ such that

$$\kappa = \sigma\left(1 - \frac{1}{\xi} - \frac{\tau}{\xi[\xi(1+\phi)-\phi]}\right)^{-1}.$$

Then,

$$\begin{aligned} M &= 2\mu^4(\sigma-1)\xi[\xi+(\xi-1)\varphi] > 0, \quad \textit{and} \\ N &= \mu^4\{\xi[\phi(\xi-2)+\xi]+ \\ &\quad +\kappa(\sigma,\xi,\phi)[2\tau+2\phi(\xi-1)+\xi]\}. \end{aligned}$$

Now, if $\mathscr{U} > 1$ exist $\rho > 0$ such that

$$\xi = \frac{1}{2}\left\{1+\frac{\phi+\sqrt{1+4\tau(1+\phi)}}{1+\phi}\right\}+\rho,$$

we have $N = \{[1+2\rho+2\phi(1+\rho)+\sqrt{1+4(1+\phi)\tau}][2(1+\phi)^2\rho^3 + (1+\phi)\rho^2(1+2\sigma+2\phi(2\sigma-1)+3\sqrt{1+4(1+\phi)\tau}+\sigma(1+\sqrt{1+4(1+\phi)\tau}+2\tau(2+2\phi+\sqrt{1+4(1+\phi)\tau}))+\rho(1+4(1+\phi)\tau+\sqrt{1+4(1+\phi)\tau}+2\sigma(1+\phi+2(1+\phi)\tau+\sqrt{1+4(1+\phi)\tau})+2\phi(2\sigma-1)\sqrt{1+4(1+\phi)\tau})]\}/\{4\rho(1+\phi)[\rho(1+\phi)+\sqrt{1+4(1+\phi)\tau}]\} > 0.$

Finally, $q_2q_1 - q_0 = \{\beta^2\mu S_3^*[\mu+\varphi(1-S_3^*)]+\eta\varphi(1-S_3^*)[2\mu(\theta S_3^*+\mu)+\theta\varphi(2-S_3^*)S_3^*+3\mu\varphi(1-S_3^*)+\varphi^2(1-S_3^*)^2]+\beta[\varphi\mu(1-S_3^*)(\mu+\varphi(1-S_3^*))+\eta S_3^* h(S_3^*)]\}/\{\eta[\mu+\varphi(1-S_3^*)]\}$ *where* $h(S) = -\theta\varphi S^2+[2\theta(\mu+\varphi)-\mu\varphi]S+\mu(\mu+\varphi) > \text{mín}\{h(0),h(1)\}$ *for any* $S \in (0,1)$ *with* $h(0) = \mu(\mu+\varphi) > 0$ *and* $h(1) = \mu^2+\theta(2\mu+\varphi) > 0$.

Thus, $q_2q_1 - q_0 > 0$. Therefore, by the Routh–Hurwitz criterion, E_3 is locally asymptotically stable.

Importantly, the local stability of equilibrium point E_i implies the unstably of equilibrium point E_j with $i \neq j \in \{1,2,3\}$. Even more, and in some cases, the very existence of the other equilibrium points. Figure 2 illustrates model (1) trajectories according to Proposition 1 results.

A switch and impulsive type model

The kth week is divided into the work week and weekend in correspondence with temporal intervals $(kT,(k+l)T)]$ and $((k+l)T,(k+1)T]$, respectively. Particularly, $T = 7$ days and $l = 4/7$ are assumed. The flows shown in Figure 1 are divided and defined in the previous temporalities according to two transference sequences:

(1) Social drinkers $\hookrightarrow$ Risk drinkers $\hookrightarrow$ Harmful drinkers, and

(2) Social drinkers $\hookrightarrow$ Binge drinkers and Risk drinkers $\hookrightarrow$ Harmful drinkers.

During weekends, the alcohol consumption prevalence is estimated from new drinkers that adopt social or binge-drinking. We assumed that input flow μN is divided into complementary rates given by $\mu\alpha N$ and $\mu(1-\alpha)N$ towards Social drinking and Binge drinking compartments, respectively. The proportion $\alpha = \alpha(t) \in (0,1)$ is represented by

$$\alpha' = \lambda(1-\alpha) - \frac{\theta S}{S+B}\alpha, \quad \alpha((k+l)T) = 1 \tag{2}$$

and inspired from Cabrera et al. (2021), Gutiérrez-Jara et al. (2022), Gutiérrez Jara and Muñoz Quezada (2022), and Gutiérrez-Jara and Saracini (2022), which modeling is associated with risk perception. The rate $\lambda \geq 0$ is a measure of the protective factors that promote a social drinking pattern through an increase of proportion α to its maximum value. Thus, in $\theta = 0$, the proportion α is equivalent to one, meaning that the first episodic alcohol consumption is with responsibility and control, typically of social drinkers. On the contrary, if $\theta > 0$, the rate λ contributes to compensating for peers' negative influence, represented by the point prevalence level. Figure 3 illustrates the interconnections among these compartments according to temporality.

Importantly, during the working week, binge drinkers do not interactions with the other compartments due to the periodicity of this pattern of consumption is exhibited by social drinkers only during the weekend that once the consumption social context is over, such as a party, binge drinkers return to initial social consumption or persist in harmful consumption. At the beginning of each work week and weekend, represented respectively by instants $t = kT$ and $t = (k+l)T$, a fraction $\delta \in (0,1)$ of binge drinkers return to social drinking, and the complementary fraction adopts harmful drinking. In turn, social drinkers adopt binge drinking a fraction $\omega \in (0,1)$. Therefore, the following model is proposed

$$
\left\{
\begin{array}{ll}
\left.\begin{array}{rcl}
S'(t) &=& \mu N - \beta S(t)\dfrac{R(t)}{N} - \mu S(t) \\[2ex]
R'(t) &=& +\beta S(t)\dfrac{R(t)}{N} - \eta R(t)\dfrac{H(t)}{N} - \mu R(t) \\[2ex]
H'(t) &=& +\eta \dfrac{R(t)}{N} H(t) - \mu H(t) \\[2ex]
B'(t) &=& 0
\end{array}\right\} & , \text{if} \quad t \in (kT,(k+l)T] \\[4ex]
\left.\begin{array}{rcl}
S(t^+) &=& (1-\omega)S(t) \\
R(t^+) &=& R(t) \\
H(t^+) &=& H(t) \\
B(t^+) &=& B(t) + \omega S(t)
\end{array}\right\} & , \text{if} \quad t = (k+l)T \\[4ex]
\left.\begin{array}{rcl}
S'(t) &=& \mu\alpha N - S(t)\left\{\dfrac{\theta S(t) + \varphi B(t)}{S(t)+B(t)}\right\} - \mu S(t) \\[2ex]
R'(t) &=& -\eta R(t)\dfrac{H(t)}{R(t)+H(t)} - \mu R(t) \\[2ex]
H'(t) &=& +\eta R(t)\dfrac{H(t)}{R(t)+H(t)} - \mu H(t) \\[2ex]
B'(t) &=& \mu(1-\alpha)N + S(t)\left\{\dfrac{\theta S(t) + \varphi B(t)}{S(t)+B(t)}\right\} - \mu B(t)
\end{array}\right\} & , \text{if} \quad t \in ((k+l)T,(k+1)T] \\[4ex]
\left.\begin{array}{rcl}
S(t^+) &=& S(t) + \delta B(t) \\
R(t^+) &=& R(t) \\
H(t^+) &=& H(t) + (1-\delta)B(t) \\
B(t^+) &=& 0
\end{array}\right\} & , \text{if} \quad t = kT
\end{array}
\right. \tag{3}
$$

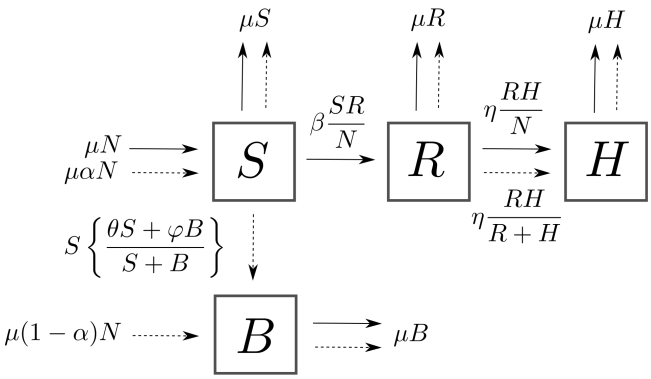


Figure 3: Progression diagram for alcoholic drinking compartmental mathematical model of switch type. The continuous lines indicate the work week, and the dashed lines the weekend flows.

such that $S(0)+R(0)+H(0)=N$ and $B(0)=0$.

RESULTS

The temporal dynamics of alcohol consumption patterns obtained from model (3) correspond to periodic trajectories that heuristically are of four types according to the persistence of the states: (i) Persistence of S and B, (ii) Persistence of S, R, and B, (iii) Persistence of S, H, and B, and (iv) all states persistence (See Figure 4).

The trajectories shape indicates the existence of maximum and minimum values of S and B during the weekends. They will tend to increase or decrease depending on the value of the parameters involved in the model (3). Thus, we define the following values

$$X_{max} = \lim_{t\to\infty} \max_{t\in I}\{X(t)\} \quad \text{and} \quad X_{min} = \lim_{t\to\infty} \min_{t\in I}\{X(t)\}$$

for $X \in \{S,B,\alpha\}$ with $I = ((k+l)T,(k+1)T]$. Note that $\alpha_{max} = 1$, and $B_{min} = 0$ when $\omega = 0$.

Therefore, How is the variation of X_{max} and X_{min} in the parametric plane $\lambda vs. \theta$ as $\rho \in \{\beta,\eta,\omega,\varphi,\delta\}$ increases? The parameters have default values according to Table 1, assuming a temporal limit $t_\infty = 350$ and $(\theta,\lambda) \in [0:0.05:1]^2$.

Peer influence on binge drinking

The alcohol consumption patterns dynamic of the weekend depends on the dynamics of the workweek, where β, η, and μ are relevant parameters. However, since μ is a parameter present in most of the equations of the model (3), the sensitivity study requires a more robust analysis, which is possible to perform using System Dynamics methodology (Aracil and Gordillo, 1997; Sterman, 2000).

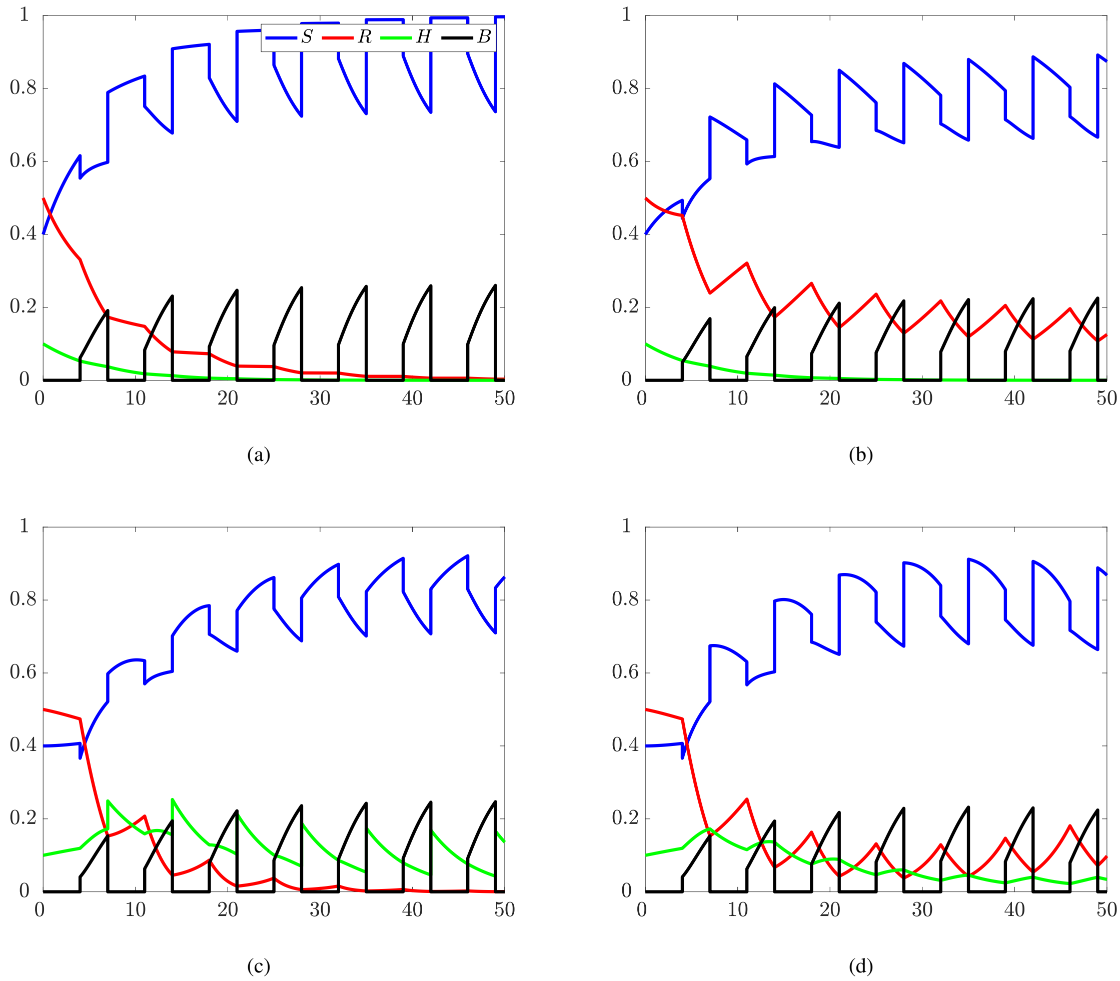


Figure 4: Temporal dynamics of model (3). We assumed $N = 1$ and initial conditions $S(0) = 0.4$, $R(0) = 0.5$, $H(0) = 0.1$ and $B(0) = 0$, with common parameters: $\mu = 0.2$, $\varphi = 0.1$, $\theta = 0.1$, $\omega = 0.1$ and $\lambda = 1$. In particularly, (a) $\beta = 0.2$, $\eta = 0.1$ and $\delta = 1$, (b) $\beta = 0.4$, $\eta = 0.1$ and $\delta = 1$, (c) $\beta = 0.6$, $\eta = 0.5$ and $\delta = 1$, and (d) $\beta = 0.6$, $\eta = 0.5$ and $\delta = 0.5$.

Effect of parameter β on the Social and Binge drinking dynamical patterns

Figure 5 shows the effect of the parameter β on values of S_{max}, S_{min}, B_{max}, and α_{min}. The red arrow indicates that the surfaces associated with the above values decrease as β increases. Importantly, for small values of θ the surfaces S_{max} and S_{min} vary, while for B_{max} it varies as θ increases. Additionally, Figure 5(d) shows a unique shape for α_{min}.

Such variations can be explained by the interactions between the state variables S and R in the model (3). As β increases, the higher the point prevalence of social drinkers by risk drinkers, which implies a decrease in social drinkers at the start of each weekend. Consequently, it decreases the relative size of S and thus the values S_{max}, S_{min} and B_{max} also.

Effect of parameter η on the Social and Binge drinking dynamical patterns

The effect of parameter η on the values of S_{max}, S_{min} and B_{max} is shown in Figure 6. The blue (respectively, red) arrow illustrates that as η increases, the surfaces S_{max} and B_{max} also increase (respectively, S_{min} decreases). Note that larger variations in S_{max} and S_{min} are obtained for small values of θ (See Fig.6(a)-(b)). However, B_{max} varies widely for large values of θ (See Fig.6(c)).

From model (3) follows that if η increases, the higher the point prevalence of risk drinkers by harmful drinkers, which implies a decrease and increase of risk drinkers and social drinkers at the start of each weekend, respectively. Consequently, the relative size of S increases, and therefore the value of S_{max} also increases. The decreasing of S_{min} and increase of B_{max} could be understood from the point prevalence of peer in equation (2) which increases due to the increase in S or θ.

Trade-off between protective and risk factors

Figure 7 shows the proportion of individuals who, in the long term, remain social drinkers at the end of each weekend, that is, those who socially start their consumption and end it the same way. In this case, its consumption pattern is not altered without succumbing to binge drinking. Consequently, the blue region is associated with the combination of high values of θ and low values of λ, indicating that a small proportion of individuals remain social drinkers at the end of each weekend. On the contrary, the red color accounts for a high proportion of individuals who remain social drin-

Parameter	Default value					
	Fig.5	Fig.6	Fig.7	Fig.8	Fig.9	Fig.10
β	0.5:0.1:0.9	0.5	0.5	0.5	0.5	0.5
η	0.5	0.3:0.1:0.7	0.5	0.3	0.3	0.3
ω	0.0	0.0	0.0	0.1:0.2:0.9	0.0	0.0
φ	0.0	0.0	0.0	0.0	0.1:0.2:0.9	0.0
δ	1.0	1.0	1.0	1.0	1.0	0.1:0.2:0.9

TABLE 1: THE PARAMETERS AND THEIR DEFAULT VALUES FOR THE SYSTEM (3) FOR VISUALIZING THE X_{max} AND X_{min} WITH $X \in \{S, B, \alpha\}$ TAKING INITIAL CONDITIONS $S(0) = 0.4$, $R(0) = 0.5$, $H(0) = 0.1$ AND $B(0) = 0$, AND THE COMMON VALUE $\mu = 0.2$.

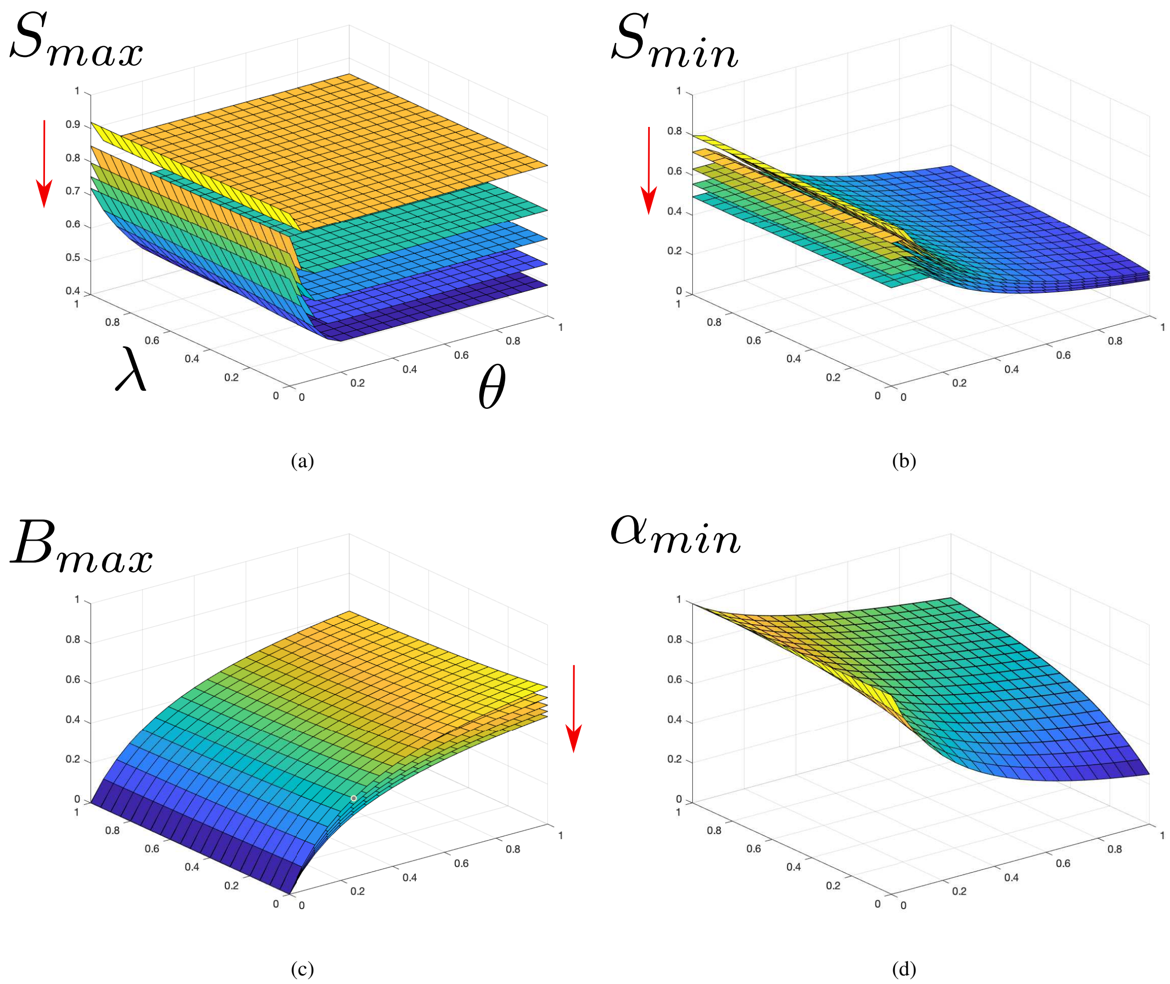


Figure 5: Influence of $\beta \in \{0.5, 0.6, 0.7, 0.8, 0.9\}$ on the surface (a) S_{max}, (b) S_{min}, (c) B_{max}, and (d) α_{min}. The red arrow indicates that the surfaces associated with the variables above decrease as β increases.

kers. To describe such an effect, the horizontal axis quantifies peer social pressure exerted by social drinkers on other social drinkers, and the vertical axis quantifies the factors of behavioral regulation, self-control, and behavior management of social drinking.

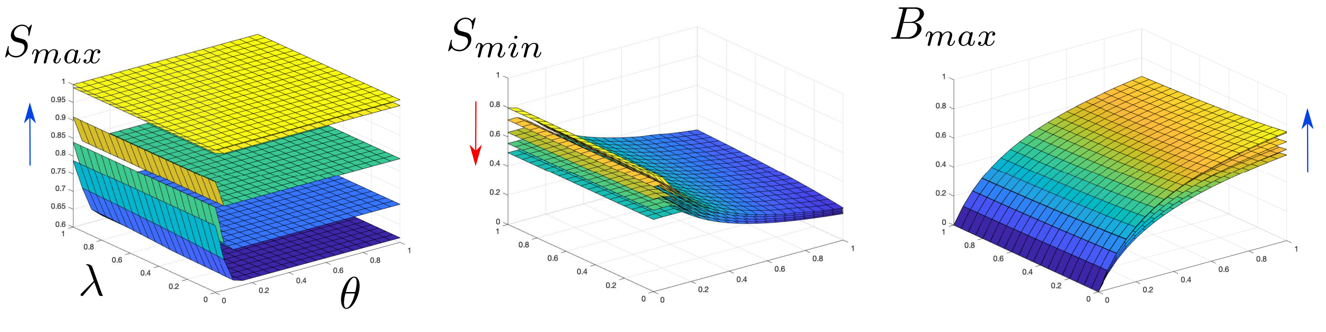
Recurrence to binge drinking

The recurrence of binge drinking is measured by the proportion $\omega \in (0, 1)$ of S. Figure 8 illustrates the effect of the parameter ω on the values of S_{max}, S_{min}, B_{max}, and α_{min}. As in the previous figures, the red arrow (respectively, blue) describes that as ω increases, the surfaces associated with the previous values decrease (increase, respectively). Note that S_{max} and S_{min} vary widely and for increasing values of θ. Conversely, B_{max} is variable for small values of θ. Additionally, as ω increases, α_{min} decreases slightly.

From the interrelations on the model (3) follows that if ω increases, a more significant proportion of social drinkers are predisposed to binge drinking at the beginning of each weekend. Thus, states B and S have high and low initial conditions, respectively. This could explains the decrease in S_{max}, S_{min} and α_{min} and the increase in B_{max}.

Dependent prevalence of binge drinkers

Model (3), with respect to model (1), does not consider the rate returning to social consumption given by φB. However, because the binge drinker label corresponds to a momentary state which is expressed in a single occasion, here during the



(a) (b) (c)

Figure 6: Influence of $\eta \in \{0.3, 0.4, 0.5, 0.6, 0.7\}$ on the surface (a) S_{max}, (b) S_{min} and (c) B_{max}. The red/blue arrow indicates that the surfaces associated with the variables above decrease/increase as η increases.

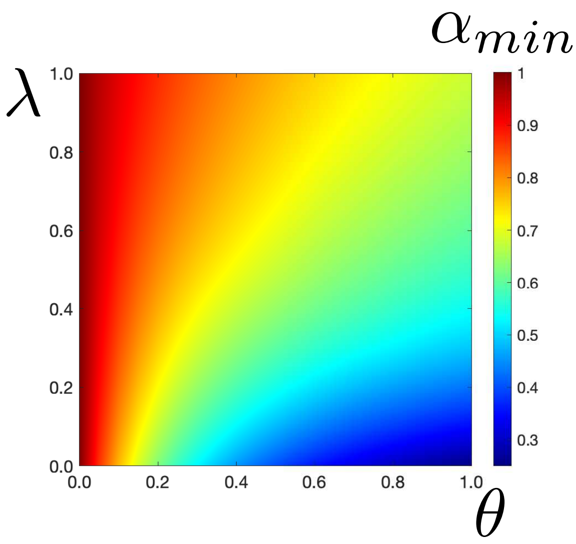


Figure 7: Value of $\alpha_{min} \in [0,1]$ according to color scale for the combination of parameters θ and λ.

weekend, such that once the drinking circumstance is over, the individual can return to social consumption (even abstemious) until the following weekend or maintain harmful drinking during the workweek.

The effect of parameter φ on the values of S_{max}, S_{min}, B_{max} and α_{min} is presented in Figure 9. As φ increases, the surfaces associated with S_{max} and S_{min} decrease. On the contrary, the surfaces associated with B_{max} and α_{min} increase when φ also increases. Note that the variability for S_{max} is highly concentrated towards small values of θ and with a rapid drop-off towards a constant value as θ increases. With respect to S_{min}, there is a homogeneous decrease in the surfaces with respect to φ increase and an increase in the surfaces associated with B_{max} and α_{min} for all the possible combinations of the θ and λ parameters. In particular, a subtle variation is observed for the α_{min} values as φ increases.

In model (3), the φ increase implies that the point prevalence of social drinkers by binge drinkers also increases. Thus, the S and B states decrease and increase, respectively. Finally, the increase of B implies a lower punctual prevalence of peers according to equation (2), where the increase of α_{min} is explained.

Harmful drinking from a sustained binge drinking

Extending binge drinking to the working week corresponds to adopting harmful consumption, which is measured by the proportion $\delta_c = 1 - \delta \in (0,1)$.

Figure 10 presents the effect of parameter δ on S_{max}, S_{min}, and B_{max} values. Considering the influence of θ, the surfaces associated with the previously mentioned values increase in all cases. For low values of θ, the S_{max} surfaces intersect. On the other hand, the surfaces S_{min} and B_{max} behave homogeneously as δ varies. Thus, increasing δ_c causes S_{max}, S_{min}, and B_{max} to decrease. The size of S and B will be low, and the drinking population will be concentrated in the harmful drinking pattern.

Discussion and Conclusions

The present research studies the temporal dynamics of different alcohol consumption patterns by formulating a compartmental mathematical model described by impulsive and change differential equations, a Filippov system. Based on the medical/health approach, dynamic "contagion" presents differences in the individual's social context. In the classical modeling of alcohol consumption, social drinkers define a class analogous to the formed by susceptible individuals in infectious-contagious disease modeling. However, we establish that social drinkers have the ability to influence the behavior of others, altering and modifying both their original patterns of alcohol consumption and that of their peers through social pressure (Buonomo and Lacitignola, 2014; Giacobbe et al., 2017; Buonomo et al., 2018; Straughan, 2019; Gutiérrez et al., 2022).

Our findings establish that depending on the value of the parameters involved in the formulated mathematical model (3), four asymptotic periodic dynamics can emerge, one more than the continuous model (1). The dynamics analysis, numerically carried out, focuses on the Social drinking and Binge drinking states during weekends and the long term, bounded by their maximum and minimum values, denoted by S_{max} and S_{min} respectively. A sensitivity analysis allowed determining the positive, negative, or neutral effect of the parameter $\rho \in \{\beta, \eta, \omega, \varphi, \delta\}$ on S_{max}, S_{min}, B_{max} and α_{min} for $(\theta, \lambda) \in [0,1]^2$. The cases where surfaces are not shown for α_{min} are due to the fact that there are no significant observable changes between surfaces. The parametric influence is

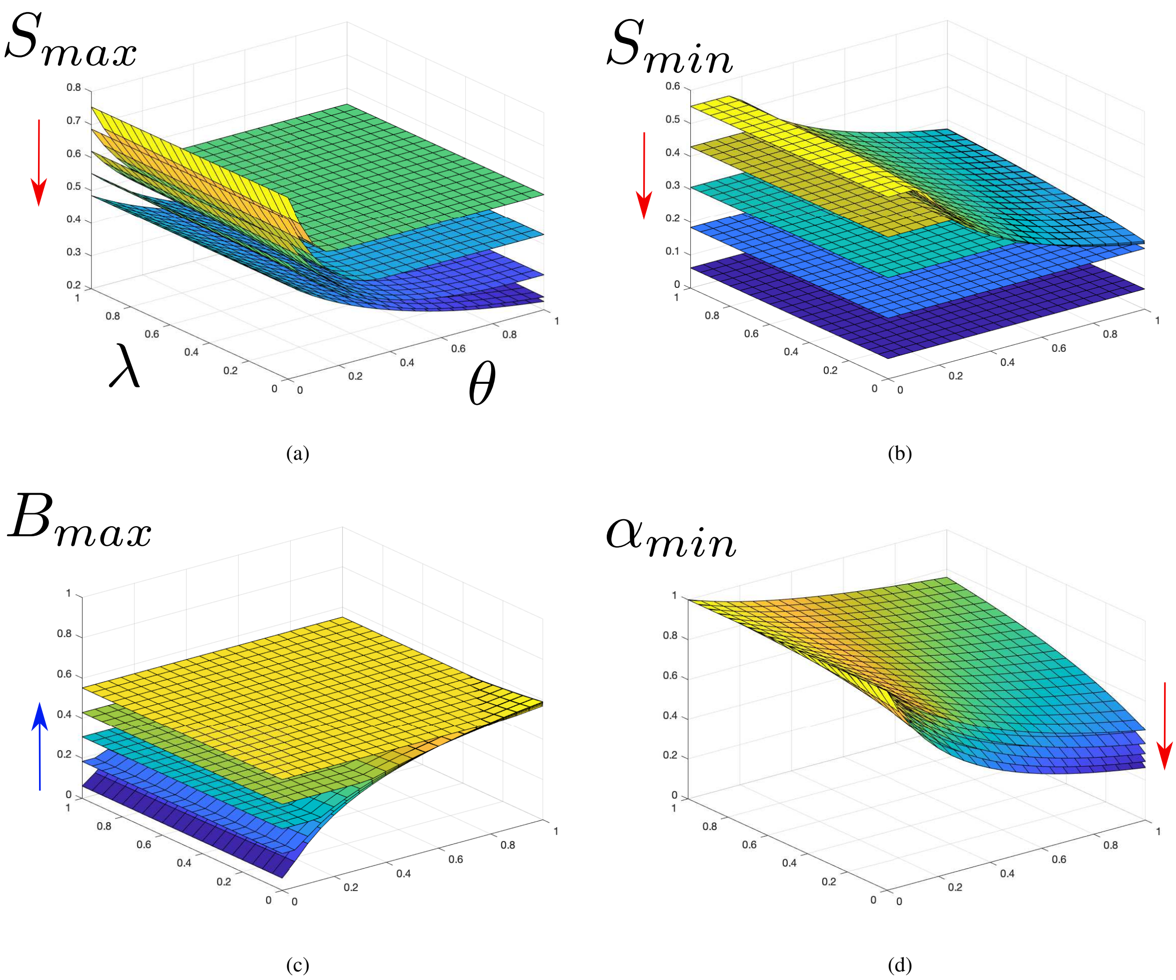


Figure 8: Influence of $\omega \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ on the surface (a) S_{max}, (b) S_{min}, (c) B_{max} and α_{min}. The red/blue arrow indicates that the surfaces associated with the variables above decrease/increase as ω increases.

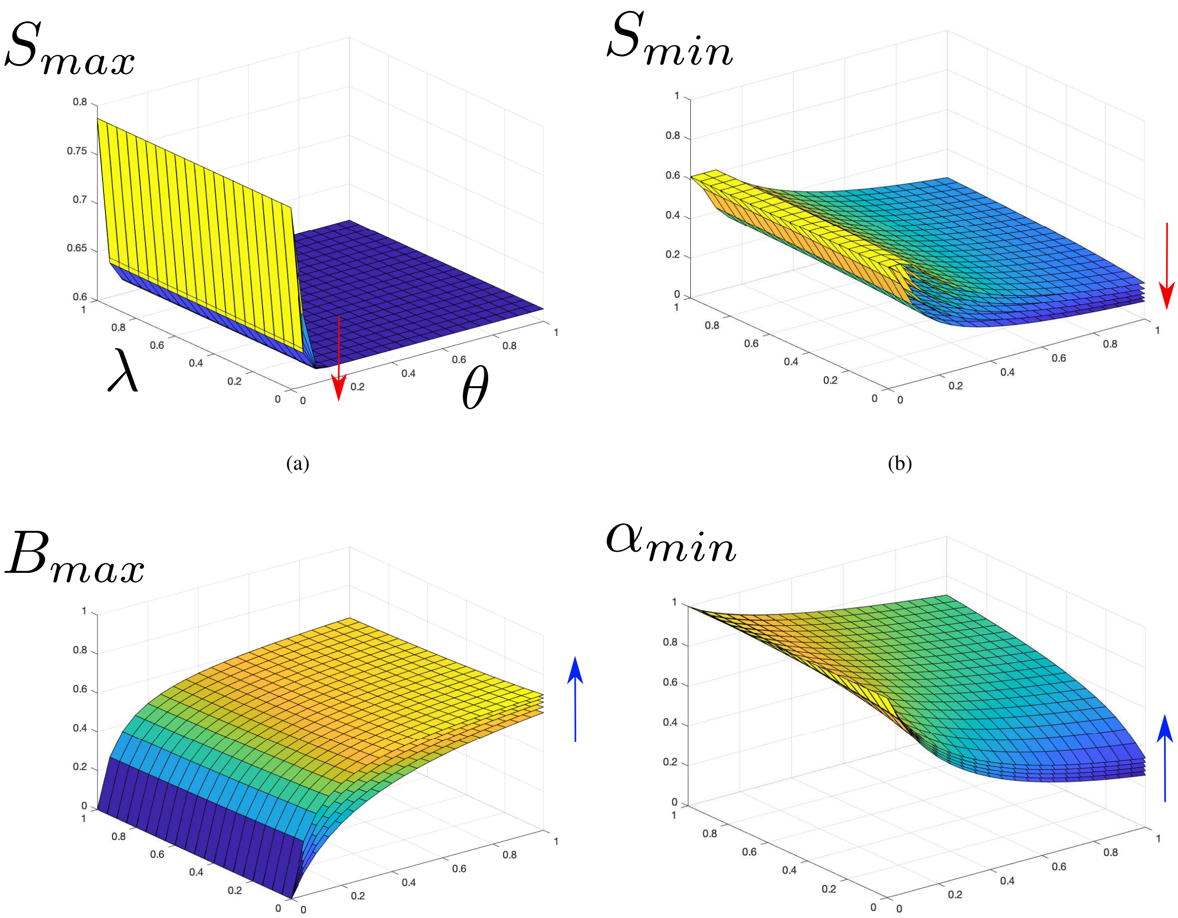
summarized in Table 2.

Parameter	Influence			
	S_{max}	S_{min}	B_{max}	α_{min}
β	$-$	$-$	$-$	0
η	$+$	$-$	$+$	0
ω	$-$	$-$	$+$	$-$
φ	$-$	$-$	$+$	$+$
δ	$+ \ (\theta \gg 0)$	$+$	$+$	0

TABLE 2: INFLUENCE OF PARAMETER $\rho \in \{\beta, \eta, \omega, \varphi, \delta\}$ ON VALUES S_{max}, S_{min}, B_{max} AND α_{min} AS INCREASES IT: POSITIVE $(+)$, NEGATIVE $(-)$ AND NEUTRAL (0).

Our main finding corresponds to the existence of a trade-off between protective and risk factors. In Figure 8, a tension between protective and risk factors are observed, evidenced by the color scale (value of α_{min}), where a large proportion of individuals maintain a pattern of social consumption when peer pressure is low. In the face of peer social pressure, we can argue that a level of self-control regulates consumption behavior and causes the maintenance of the original pattern, social consumption. Furthermore, it is possible to determine thresholds of support and resistance to peer social pressure that is exerted in contexts of alcohol consumption. When peer pressure (θ) is greater than self-control (λ), the original consumption pattern may alter and establish a start toward new consumption patterns.

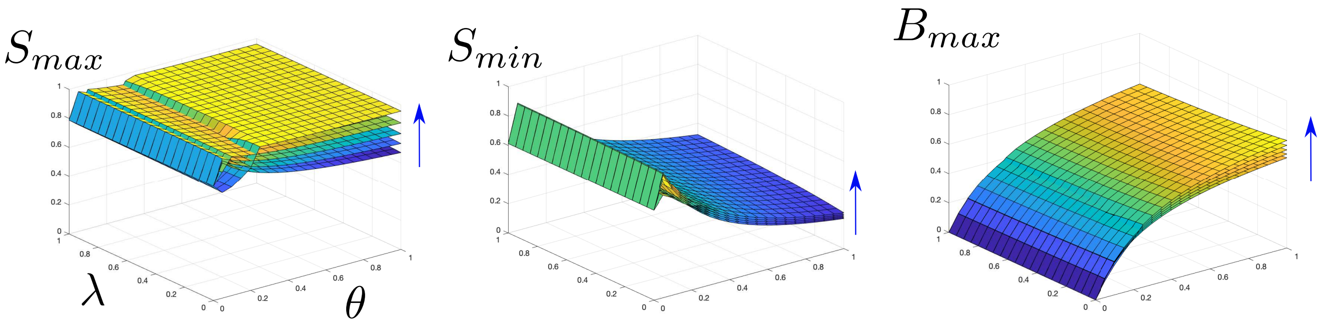
In this study, and as a limitation, only the effect caused by peer social pressure on consumption social patterns is considered, both positively and negatively, understanding that there may be other variables that also affect it, such as school and family (Rojas-Jara and Leiva-Vásquez, 2018). The latter is transformed into a proactive drift towards new studies on factors capable of altering alcohol consumption behaviors.



(a) (b)

(c) (d)

Figure 9: Influence of $\varphi \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ on the surface (a) S_{max}, (b) S_{min}, (c) B_{max} and (d) α_{min}. The red/blue arrow indicates the surfaces associated with the variables above decrease/increase as φ increases.



(a) (b) (c)

Figure 10: Influence of $\delta \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ on the surface (a) S_{max}, (b) S_{min} and (c) B_{max}. The blue arrow indicates the surfaces associated with the variables above increase as δ increases.

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