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Departamento de Matemática
Facultad de Ciencias Naturales, Matemática y Medio Ambiente

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TECNOLÓGICA METROPOLITANA

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Journal of Mathematical Modelling of Biological Systems

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Editorial

ALGUNOS ASPECTOS ONTOLÓGICOS Y EPISTÉMICOS PARA LA MODELIZACIÓN EN INVESTIGACIÓN INTERDISCIPLINAR

Fernando Córdova-Lepe
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Un saber generado por investigación interdisciplinar mediante modelización matemática, en su capa más exterior, no es conocimiento calificable de matemático; al situarlo en la balanza fáctico-formal esta se inclina hacia el platillo disciplinar, el atingente al realismo. Además las disciplinas no matemáticas concurrentes cuentan a su favor con varios aspectos prácticos, e. g., aventajan en capacidad de comunicar al gran público sus avances y conclusiones respecto de un fenómeno bajo estudio. En efecto, al indagar en problemáticas propias de ciencias no del todo matematizadas (como la biología) el modelizador ha de reconocer que el protagonismo se ubica en lo fáctico, el contexto que define: la intención del modelo, en gran medida los utensilios y el relato conclusivo; esto último a través de registros figurativos y un lenguaje cercano al natural, más cómodo y de alcance mayor, digamos donde Mathema no llega.

Al observar la presencia de las matemáticas en experiencias de investigación interdisciplinar resalta principalmente que estas tienen un rol metodológico. En este sentido, es importante que los agentes modeladores, en favor de su arte, conozcan y articulen las diferencias ontológicas y epistemológicas existentes entre los elementos propios de la matemática, que dan cuenta de su ser y potencial, con las correspondientes formas y métodos de validación empíricos de las ciencias naturales. El configurarse una panorámica y nivel de pronunciamiento sobre estas disimilitudes, es un mínimo exigible a un modelador matemático; es la pretensión de esta editorial, sin más afán de compartir un informal asomo al tema.

EL ESPACIO INTERDISCIPLINAR

La investigación en matemáticas conlleva una presunción, otorgarle a los elementos en análisis cierto estatus de realidad objetiva. Ante un desafío de la especialidad, el hacedor de matemática, con sentido estratégico, combina y le da existencia a sus entidades ideales por medio de reglas de deducción prestablecidas, un tipo de juego híper regulado y complejo, por lo general, socialmente inocuo. A veces, un dramático soliloquio en un conflicto autoimpuesto o presentado por pares de la cofradía temática. Sin embargo, cuando el desafío-problema no es entidad abstracta y matemática, sino que está mediado por una disciplina de orden fáctico, para el matemático aplicado establecer su juego acostumbrado pasa por sumar una complejidad diferente, pues deja de ser preestablecido y totalizador en cuanto a tipos de objetos, reglas y consecuencias. Además, suelen emerger límites a las expectativas del equipo y aristas de subjetividad, potencialmente paralizantes para el modelizador novato.

Dentro de un equipo de investigación interdisciplinar, a la presentación de una situación problema en el lenguaje de una ciencia de contexto no matematizada y ad hoc al fenómeno bajo observación, debe seguir un proceso de adaptación y reordenamiento de perspectivas y un consenso del plano de la comunicación formal del equipo. Nos referimos a una serie de elementos técnicos y también prácticos que de quedar bien o mal instalados, pueden llegar a determinar el éxito o fracaso del equipo. Aunque, siendo estos elementos importantes, existen otros de profundidad filosófica, por ejemplo, el rol epistemológico reservado a la matemática, también ineludibles para el modelador especializado que busca entender su quehacer.

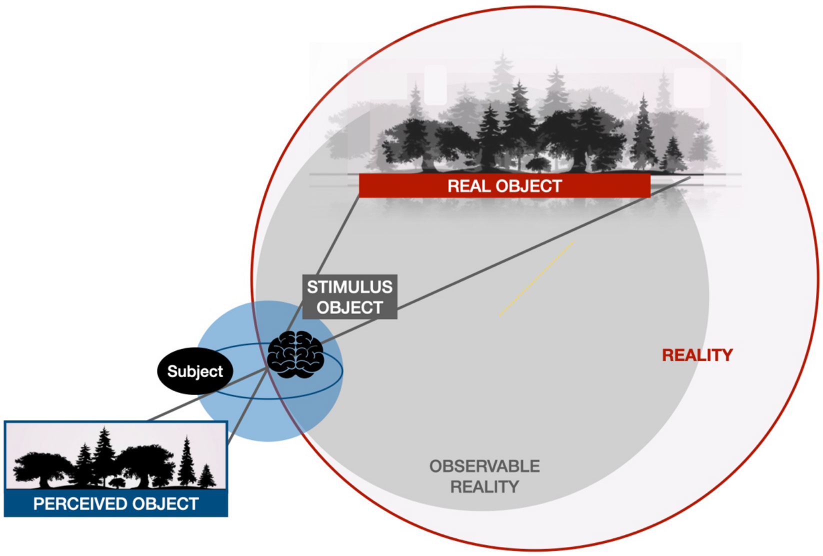

Figura 1. Relación sujeto y sus objetos asociados: objeto-real (cosa en sí), objeto-estímulo (como unidad sensorial) y objeto-percibido (mediado y registrado por la consciencia). Existe una perspectiva intencionada sobre el objeto-real que está en coevolución con la historia de vida del objeto-percibido en el sujeto.

En contraste al espacio de trabajo idealizado y convencionalmente determinado con que cuenta un laborioso instalador de teoremas con sus respectivas pruebas lógico-matemáticas, el modelador recibe y reacciona ante un fenómeno-problema (natural o social) a través de procesos helicoidales de abstracción, análisis e interpretación. En estos, las sucesivas puestas en símbolos y relaciones matemáticas de los configurantes son actos inseparables de sus pretendidas transducciones semánticas. Comenzará por lo ya conocido de estas realidades y por el alcance de las preguntas de investigación del equipo investigador. Un modelo, como posibilidad de objeto matemático, llevará la indeleble y decantada marca de lo que son las percepciones contingentes, históricas y anhelos que los investigadores concurrentes tienen sobre el trozo de realidad bajo la lupa. En su tarea, el modelador para optimizar su rol en la investigación interdisciplinar, está llamado a explorar la intersubjetividad en pro de una comunicación efectiva. Notemos que en este hacer científico, está en valor quien se adapta a la idea de cognición compartida y al logro permanente de consensos, pues resulta esencial para la renovación de las ideas. La investigación científica vía equipos interdisciplinarios es un acto dialógico, que parte por el reconocimiento de la percepción y las formas de las otredades disciplinares en cuánto a la naturaleza de los modos de situar los objetos de investigación y los métodos de elaboración de certezas involucrados.

DIFERENCIACIÓN ONTOLÓGICA

Un sujeto intencionado frente a un espacio de observación sensible o psíquico busca el reconocer o dar forma a un pedazo o trama de tal existencia. Así, ante el emerger de unas primeras líneas de borde que aprecie de interés, tiende a desplegar su facultad biológica de obtener un recorte más estable; aplica sus tijeras mentales para generar una impresión con la que posteriormente juzgará adaptativamente sus experiencias futuras. Si sistematiza, logra fijar y penetrar en lo que a otros les es efímero e inexpugnable, capturando cierta unidad identificable, e. g., lo que es la idea de bosque desde el paisaje, ver Figura 1. Así, el sujeto ha dado forma a lo que denominamos objeto percibido; un recorte por lo general plástico y dinámico. Este objeto percibido tiene un conjunto de posibilidades de realización, o estados, en conexión con otros objetos y el mismo sujeto, y se presenta siendo parte de sistemas de objetos, una suerte de fotografías identitarias factibles.

Los estados de un objeto son en general inestables, proclives a cambios por estímulos naturales o por intervención. Unos estados, los causales, llevan necesariamente o hacen pasar al objeto a un estado sucesor, el estado efecto, y no así a otros que son independientes. En este sentido, si un estado es una fotografía, los estados derivados en orden temporal conforman un proceso, es decir, definen un trayectoria. Así, visualizamos historias de vida de los objetos como filmes cinematográficos factibles, lo que en sistemas dinámicos llamamos órbitas. Los determinantes de la trayectoria de un objeto percibido, los desarrolladores de su guion específico, están dados por la acción conjunta de conexiones subyacentes a la estructura de la realidad-experiencia, una arquitectura de relaciones (leyes), que para no caer en rigidez determinista, podemos también admitir vibrante.

Hay necesidad de identificar al interior del espacio de existencia del objeto y su historia (realidad) el subespacio de sus representaciones. En el mundo de los hechos, lugar de interacción y transformación de los objetos en sí, distinguir el lugar que tenemos para referenciarlos y pensarlos; sea con óptica de disciplina de contexto o la del idealismo matemático. En este sentido, consideramos adecuado reservar para los objetos y hechos de la realidad el principio de identidad concreta de la ontología dialéctica, que permite pensar las trayectorias del objeto-percibido como la unidad que se diferencia y desdobla. Mientras que reservamos el principio de identidad abstracta de la lógica matemática para los objetos y sistemas referenciales ideales. Al respecto, puntualicemos que un objeto percibido es unidad en cuanto lo que es observado es alguna característica general y se desdobla por aquellas características más particulares, una separación que un objeto matemático afortunadamente no presenta. Notemos que “en el proceso de la cognición idealizar y simplificar la realidad, en ciertas condiciones, no sólo resulta posible, sino incluso necesario” (Identidad en Diccionario Filosófico, Rosental-Iudin). Quien modela debe estar consciente que la dualidad abstracto-concreto es una llave permanente para entender y desarrollar los actos tendientes a generar conocimiento de los objetos en algún aspecto específico deseado, en que responder ¿qué se abstrae y qué no del objeto-percibido? es la vía para llegar a alcanzar saberes respecto de lo concreto. Destaquemos también que a un modelador le viene cómodo tener a la propia lógica como modelo abstracto de las relaciones causa-efecto del lenguaje natural.

¿Qué bases tenemos para sostener la esperanza de conocer, de alcanzar conocimiento a través de un proceso de modelización? ¿Dónde radica nuestra confianza? Al respecto, Wittgenstein afirma que el lenguaje es un modelo del mundo, que las cosas del lenguaje son las proposiciones, que las del mundo son los hechos (sistemas de objetos) y que existe una correspondencia entre hechos y proposiciones que refleja en la estructura del lenguaje la del mundo. La matemática, en cuanto lenguaje formal, que permite la argumentación para la derivación argumental, es un camino viable, aspecta muy bien para revelarnos tan íntima arquitectura del mundo. Así, frente a la primera pregunta de toda teoría del conocimiento: ¿Es posible conocer? La respuesta, confiando en el autor del *Tratado lógico-filosófico* es afirmativa.

La Figura 2 muestra un paisaje de la realidad en que destaca un arcoíris, el cual en cuanto a recorte de cierto sujeto es un sistema que conecta con la lluvia y el sol, pero ajeno al objeto bosque. Muy probablemente en la experiencia de tal sujeto hay registro de otros arcoíris sin la presencia de bosques. Sin embargo, un sujeto que nunca ha salido de su aldea situada en plena foresta no necesariamente dejaría a éste fuera del registro. Esto sugiere que la experiencia colectiva, como la de un equipo interdisciplinar, suma a las posibilidades de reconocer las conexiones esenciales de aquellas meramente contingentes. Hacemos notar que esta propiedad de la consciencia, de desconectar o conectar percepciones, es un aspecto clave en la investigación vía modelización.

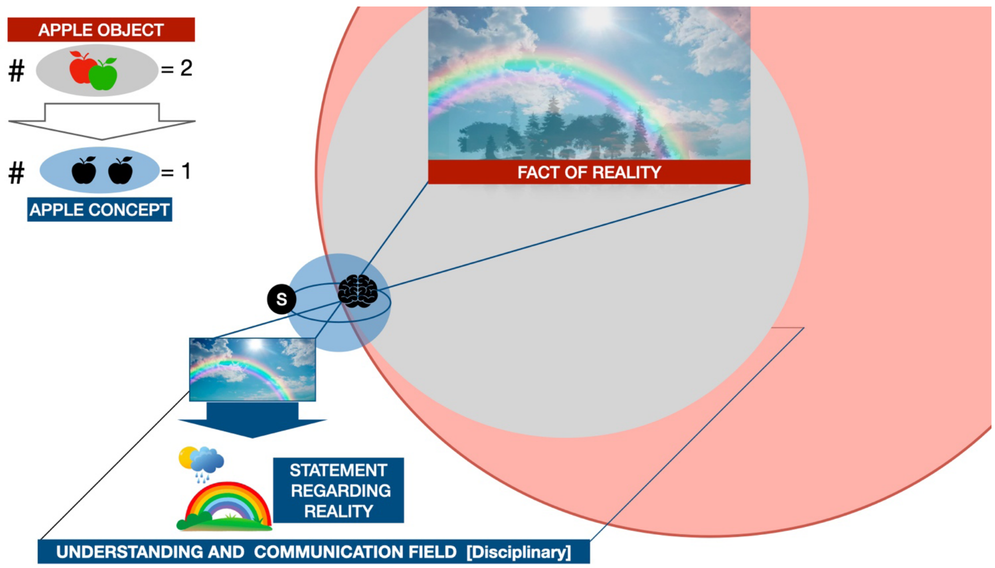

Figura 2. Una comunidad disciplinar registra y proyecta su experiencia sobre rasgos específicos de interés común de los hechos sobre el plano de intelección y comunicación vía conceptos y enunciados. El concepto debe ser único, no hay dos conceptos de manzana, pero sí pares de manzanas. El enunciado conecta conceptos y se juzga como verdadero (o falso) por experimentación.

Dado un hecho existirán tantas perspectivas como sujetos observantes, pero si estos forman una cultura pues tienen regular afán investigativo e intención ontológica semejante, digamos que comparten el set de características de los objetos en que se enfocan y hacen uso de extensiones consensuadas de los sentidos (mismas tijeras), ellos están dando cuerpo a una disciplina de contexto. Estas comunidades científicas hacen registro de la experiencia común y de las proyecciones que determinan (lo representado), lo que en la Figura 2 etiquetamos como plano de la intelección y la comunicación disciplinar. Entonces, un mismo hecho, en cuanto a conexión (o sistema) de objetos de la realidad, puede presentar tantas variantes en cuanto a objeto-estímulo y objeto-percibido, como disciplinas estén interesadas en el hecho en cuestión.

El plano disciplinar es el lugar donde nace y se sitúa cada nuevo concepto científico, modo de rotular a una familia de objetos-percibidos que tienen un patrón reconocido por la experiencia colectiva. Los conceptos, para la consistencia lógica del trabajo con ellos, han de ser únicos (presentar consenso). Con frecuencia ocurre que la palabra asociada con el concepto coincide con la palabra usada para cualquiera de sus representados. En la esquina superior izquierda de la Figura 2 representamos esto, ya que como objetos de la realidad pueden existir dos manzanas, pero nunca dos conceptos manzana. Lo anterior no es dilema para un matemático, pues ante la pregunta por la cardinalidad del conjunto {1/2 ; 0, 5} no duda en responder que es uno, un síngleton, ya que en matemática los objetos son siempre sólo conceptos. Sólo si quien percibe estuviera enfocado en la grafía, esto es, en el significante, modo que está por fuera de la intención matemática, en razón diría que dicha cardinalidad es dos.

Llegamos a un punto en que es necesario distinguir entre los planos de intelección involucrados en el trabajo interdisciplinar. Están los planos de contexto, siendo aquellos en que sus conceptos están directamente ligados con los objetos fácticos de interés de la disciplina. En estos planos (o espacios proyectivos) la unidad fundamental de conocimiento y comunicación es el enunciado, una afirmación que conecta conceptos y que se establece con la intención de justificar el por qué un hecho ocurre o no. Por otro lado, está el plano matemático, en que los conceptos se liberan de su amarre con los objetos de la realidad y pasan a tener un interés en sí mismos, en los conceptos y las relaciones consistentes entre estos; es decir, se convierten en objetos de estudio, objetos de un tipo diferente, vacíos de realidad concreta, técnicamente denominados objetos ideales. En este plano también tenemos los enunciados, que para distinguirlos preferimos llamar proposiciones. En cuanto ambos tipos de planos, el disciplinar y el matemático, involucran lenguaje podemos recurrir al muy conocido triángulo semiótico para puntualizar diferencias entre enunciados y proposiciones que un modelador debe tener en permanente consideración. Como indica la asociación vía colores en la Figura 3, el plano disciplinar está superlativamente orientado a la relación de sus signos con sus referentes, a lo semántico, que linda con el movedizo terreno de la verdad. Por otro lado, en el espacio matemático la orientación del quehacer da cuenta de la consistencia y de las derivaciones lógicas de las proposiciones entre sí; esto es, predomina una perspectiva sintáctica. En este sentido, es posible establecer un paralelismo contrastante entre enunciados y proposiciones, mientras los primeros son objetos conceptuales con semántica exterior, i. e., con referencia a los hechos, los segundos hacen referencia a objetos ideales con una semántica interna y en que la correspondencia buscada es con la razón, a lo que nos referiremos como corrección del pensamiento. Así, la tarea última del modelador en equipos interdisciplinarios es concluir desde el modelo proposiciones derivadas por argumentos correctos, las que interpretadas y en la forma de enunciados, no sólo sean validas, sino que aspiren a ser verdaderas.

DIFERENCIACIÓN EPISTEMOLÓGICA

Respecto de la milenaria disputa de si es la razón o la experiencia la génesis de nuestros saberes respecto del mundo, un modelador en contextos interdisciplinarios alguna claridad habrá de construir para efectos de la convivencia que ha de sostener entre los métodos de su disciplina y los clásicos de las ciencias fácticas. Por ejemplo, en el método hipotético deductivo (MHD) que a grandes bloques asume el flujo: observación, hipótesis y consecuencias contrastables, ¿dónde se ubica el aporte de una modelización matemática? Tener alguna luz es una necesidad que surgirá en la interacción interdisciplinar, esta probablemente se presentará al definir los roles y al evaluar el alcance del aporte formalista.

En las ciencias fácticas el investigador está interesado en consolidar o en innovar en respuestas a ciertas preguntas de investigación y, desde el análisis sobre observaciones previas, levantar una hipótesis, una conjetura contexto-causas-efectos que explique el fenómeno de interés. La hipótesis normalmente se prueba a través de la implementación de diseños experimentales específicos. ¿Qué hace un modelo ante una hipótesis? ¿Un modelo matemático puede llegar a rechazar o aceptar una hipótesis? ¿Las proposiciones matemáticas derivadas tienen con claridad, por interpretación, un relato de verdad como enunciados sobre la realidad?

Al respecto, sobre las contrastaciones leemos: "En este Método (el MHD) las contrastaciones deben ser experimentales. Y deben ser contundentes, en el sentido de que deben decirnos claramente si las consecuencias deducidas de la hipótesis se dan en la realidad o no" (*Sobre un concepto histórico de ciencia*, Carlos Pérez).

Como ya fue observado, en el trabajo en equipos interdisciplinares concurren el plano proyectivo teórico (el disciplinar) y el plano matemático. Notemos que una hipótesis científica es un enunciado del plano de contexto, por lo que no es un hecho y, en principio, tampoco una proposición matemática, y sólo se vincula con los hechos por referencia. De modo que, entrando al terreno de la verdad, observamos que es en este plano disciplinar donde los enunciados pueden llegar a ser verdaderos o falsos, ya que en el de la realidad, no hacen sentido esos calificativos, pues los hechos ocurren o no (*se dan* o *no se dan*). Ahora, respecto del plano de la matemática, es mejor reservarnos otros términos, pues para la comunicación de grupo es prudente referirnos a proposiciones, destacando las que tienen una demostración a distinguir entre el pensamiento deductivo correcto o incorrecto. Así lo deseado es alcanzar enunciados hipotéticos que tienen un correlato matemático como proposición correcta en el paso del antecedente al consecuente y, que además interpretadas pasen la prueba o contrastación empírica para convertirse en verdaderas. Aunque esto último, es sobre la base de resistir un testeo permanente de no falsedad en el campo experimental.

	Proposiciones Matemáticas Math*ematical Propositions*	
	Correctas *Correct*	**Incorrectas** *Incorrect*
Enunciados **V** *Statements* ***T***	SÍ *YES*	NO
Disciplinares **F** *Disciplinaries*	NO	NO

Notemos que, una hipótesis es una apuesta de realidad, es aquel enunciado con algún grado de fundamento y, además, contrastable que, presuponiendo ciertos hechos afirma que necesariamente se deriva otro hecho, generalmente relacionado con la especificación para un objeto-percibido de un estado o un tipo de estados dentro de los factibles. Entonces, si bien el MHD nos habla del contraste empírico a través de la experimentación, también existe el contraste de racionalidad y por ahí surge una primera oportunidad de la modelización matemática, testeando si la hipótesis es lógicamente consistente. Otra posibilidad es modelando las contrastaciones empíricas (v. g., no posibles de implementar), como experimentos mentales, chequeando que sus conclusiones analíticas van (admiten la interpretación semántica) en la dirección de lo que afirma la hipótesis. Sin embargo, un enunciado como la hipótesis, con su generalidad, aún siendo lógicamente consistente, puede llegar a ser un enunciado menos verosímil, nunca se está seguro que aparezca un experimento que le contradiga. Así, ya que la verdad de una hipótesis consistente se juega su verosimilitud en el control empírico, bajando las expectativas los modelos sólo suman o restan argumentos o soporte para su formulación.

Los equipos científicos monodisciplinares suelen estructurar o guiar su plan de trabajo a través de una pregunta de investigación, lo mismo ocurre en los interdisciplinares. ¿Qué tipo de modelo es el adecuado para esta pregunta? ¿Cuáles son las transferencias legítimas desde lo representado hacia el modelo representante (abstracción), e inversamente (interpretación)? ¿Cuáles son los elementos mínimos, primeros y necesarios de considerar, para llegar a construir un modelo con algunas pretensiones analógicas respecto de fenómenos de la realidad con posibilidades de aportar a una respuesta de la pregunta de investigación? Vale tener claridad respecto de las distinción ontológica entre objetos y hechos de la realidad, el lenguaje como articulador de conceptos a través de las proposiciones y el conjunto de objetos ideales y relaciones formales, lo matemático que aspira a convertirse en modelo.

DESPEDIDA

Al modelar no sólo debemos estar conscientes de las diferencias entre contexto y matemática respecto de objetos, proposiciones declarativas y métodos involucrados, hay muchos otros aspectos con trazas filosóficas que un modelador puede explorar para una mejor comprensión de su labor. Un enorme espacio que tienta al afán exploratorio, es el sentido de valor que involucra la tarea, sea el de uso y, por qué no, también el estético, pero esta editorial ya se ha alargado demasiado.

Finalizo apuntando que ante una pregunta de investigación, su hipótesis asociada y un nivel de precisión de la respuesta, a parte de lo ya expuesto, es útil y bello que el modelador aspire a construcciones que realicen la tarea en la forma más simple posible, vale aquí para esta y otras temáticas recomendar el libro *Deja a la estructura hablar: modelización y análisis de sistemas naturales, sociales y socioecológicos*, del dr. Rodrigo Ramos.

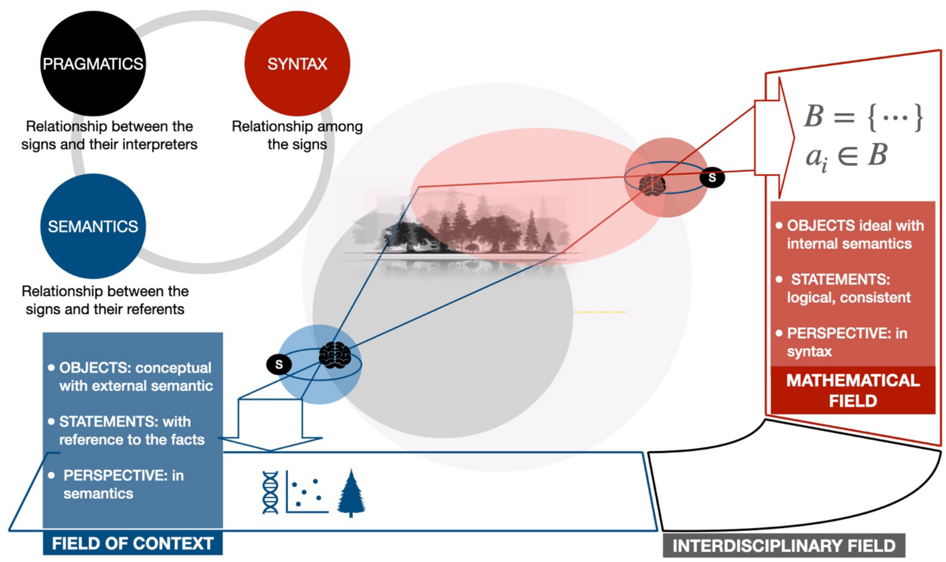

Figura 3. En el plano contexto, el enunciado tiene significación en los hechos; su semántica es exterior, se da o no en sus referentes reales. En cambio, en las proposiciones matemáticas, los referentes son solo conceptos, objetos-ideales, una semántica interna; resalta la sintaxis, la relación de los signos entre sí. La interacción va definiendo por adaptación el plano interdisciplinar de comunicación en el que conviven enunciados de contexto y proposiciones matemáticas. En cuanto a la pragmática, tercer vértice semiótico, es preocupación de otros planos imaginables, e. g., los conectados con la psicología o las ciencias de la educación.

Editorial

SOME ONTOLOGICAL AND EPISTEMIC ASPECTS FOR MODELING IN INTERDISCIPLINARY RESEARCH

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Knowledge generated by interdisciplinary research through mathematical modeling, in its outermost layer, is not knowledge that can be classified as mathematical. When placing it in the factual-formal balance, it leans towards the disciplinary dish, the one related to realism. In addition, the concurrent non-mathematical disciplines have several practical aspects in their favor, e. g., they have an advantage in the ability to communicate to the general public their progress and conclusions regarding a phenomenon under study. Indeed, when inquiring into problems typical of sciences that are not completely mathematized - such as biology - the modelers should recognize that the leading role is located in the factual side, i.e., the context that defines: the intention of the model; to a large extent the utensils; and the concluding story. The latter is performed by means of figurative registers and a language close to natural, which should be more comfortable and have a greater scope, let's say where *máthēma* does not reach.
Observing the presence of mathematics in interdisciplinary research experiences mainly highlights that it has a methodological role. In this sense, it is important that modeling agents, in favor of their art, know and articulate the ontological and epistemological differences existing between the mathematical elements, characterized by their being and potential, with the corresponding forms and methods of empirical validation of the natural sciences. Configuring an overview and level of pronouncement on these dissimilarities is a minimum required from a mathematical modeler, which is the goal of the present editorial, with no further desire than sharing an informal glimpse of the topic.

THE INTERDISCIPLINARY SPACE

Research in mathematics entails a presumption, granting the elements under analysis a certain status of objective reality. Faced with a challenge from the specialty, mathematicians, with a strategic sense, combine and create their ideal entities by means of pre-established deduction rules. It is a type of hyper-regulated and complex game, generally socially innocuous. Sometimes, it is a dramatic soliloquy in a self-imposed conflict or presented by thematic brotherhood peers. However, when the challenge-problem is not an abstract and mathematical entity, it is mediated by a discipline of a factual order. The applied mathematicians can establish their usual game by adding different complexities, since they are no longer pre-established and totalizing types of objects, rules and consequences. In addition, limits to the expectations of the teams and edges of subjectivity tend to emerge, potentially paralyzing for novice modelers.

Within interdisciplinary research teams, the presentation of a problem situation in the language of a non-mathematized science context and *ad hoc* the phenomenon under observation should be followed by a process of adaptation and reordering of perspectives and a consensus on the field of formal team communication. We are referring to a series of technical and also practical elements that, if well or badly established, can determine the success or failure of the teams. Although these elements are important, there are others of philosophical depth-e. g., the epistemological role reserved for mathematics-also unavoidable for specialized modelers who seek to understand their work.

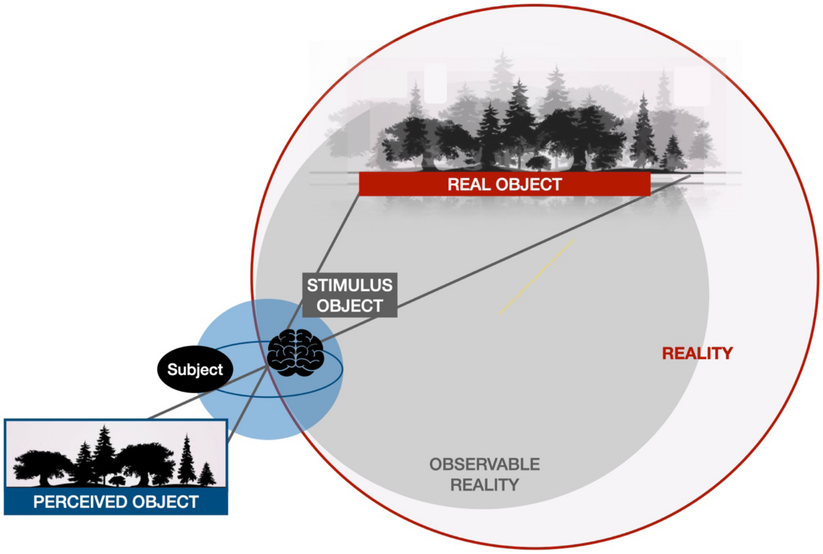

Figure 1. Relationship between subjects and their associated objects: real object (thing in itself); stimulus object (as a sensory unit); and perceived object (mediated and registered by consciousness). There is an intentional perspective in the subject with respect to the real object that is in coevolution with the life history of the perceived object.

In contrast to the idealized and conventionally determined workspace available to laborious installers of theorems with their respective logical-mathematical proofs, modelers receive and react to a phenomenon-problem (natural or social) through helical processes of abstraction, analysis and interpretation. In these processes, the successive putting into symbols and mathematical relationships of the shaping elements are actions that cannot be separated from their alleged semantic transductions. Modelers will begin with what is already known about these realities and with the scope of the research questions of the research teams. A model, as a possibility of a mathematical object, will bear the indelible and distinguished mark of the contingent and historical perceptions and desires that researchers have about the piece of reality under magnifying glass.

In their tasks, the modelers are called to explore intersubjectivity in favor of effective communication in order to optimize their role in interdisciplinary research. It is worth noting that, in this scientific practice, those who adapt to the idea of shared cognition and the permanent achievement of consensus are valuable, since these aspects are essential for the renewal of ideas. Scientific research via interdisciplinary teams is a dialogical action. It begins with the recognition of perception and the forms of disciplinary otherness in terms of the ways of situating the research objects and the methods for elaborating certainties.

ONTOLOGICAL DIFFERENTIATION

Determined individuals facing sensitive or psychic observation spaces seek to recognize or shape pieces or plots of such existences. Thus, in view of the emergence of some first border lines that they consider of interest, they tend to deploy their biological faculty to obtain a more stable cutout. They make use of their mental 'scissors' to generate an impression with which they will later adaptively judge their future experiences. If they systematize, then they can focus on and penetrate what is ephemeral and impregnable to others, capturing a certain identifiable unity, e. g., the idea of forest from the landscape (Figure 1). Thus, the individuals have given shape to what we call the perceived object, which is generally a plastic and dynamic cutout. This perceived object has a set of functioning possibilities or states, in connection with other objects and the same individuals. It appears as part of object systems, a sort of feasible identity photographs.

The states of an object are generally unstable, prone to changes by natural stimuli or by intervention. Some states, such as the causal ones, necessarily lead or make the objects pass to a successor state, i.e., the effect state, and not others that are independent. In this sense, if a state is a photograph, the states derived in temporal order create a process, i.e., they define a trajectory. Thus, it is possible to observe the life histories of the objects as feasible cinematographic films, which in dynamical systems we call orbits. In the determinants of the trajectory of a perceived object, the developers of its specific script are given by the joint action of connections underlying the structure of reality-experience, an architecture of relationships (laws), which, in order not to fall into deterministic rigidity, can also be considered vibrant.

There is a need to determine the inner existence space of the object and its history (reality), the subspace of its representations. In the world of facts, place of interaction and transformation of the objects, we should determine the place we have to reference and think about them; be it from the optics of the field of context or that of mathematical idealism. In this sense, we consider it appropriate to reserve the principle of concrete identity of dialectical ontology for the objects and facts of reality, thus allowing us to think about the trajectories of the perceived object as the unit that differentiates and unfolds. On the other hand, we reserve the principle of abstract identity of mathematical logic for ideal referential objects and systems. In this regard, it should be highlighted that a perceived object is a unit, whereas what is observed is some general characteristic unfolded by those more particular characteristics, a separation that, fortunately, is not observed in a mathematical object. It should be noted that "in the process of cognition, idealizing and simplifying reality, under certain conditions, is not only possible, but even necessary" (Identity in *Diccionario Filosófico* [Philosophical Dictionary], Rosental-Iudin).

Who models should be aware that the abstract-concrete duality is a permanent key to understand and perform the actions tending to generate knowledge about the objects in some specific desired aspect, in which answering what is abstracted and what is not from the perceived object is the way to reach knowledge regarding the concrete dimension. It is also worth emphasizing that it is convenient for a modeler to have logic itself as an abstract model of the cause-effect relationships in natural language.

What bases do we have to sustain the hope of knowing, reaching knowledge through a modeling process? Where does our trust lie? In this regard, Wittgenstein affirms that language is a model of the world, that the things of language are propositions, that those of the world are facts (object systems) and that there is a correspondence between facts and propositions reflected in the structure of the language of the world. Mathematics, as a formal language, which allows argumentation for the argumentative derivation, is a viable way. It is adequate to reveal such an intimate architecture of the world to us. Thus, facing the first question of any theory of knowledge: Is it possible to know? The answer, trusting the author of the Logical-philosophical Treatise, is affirmative.

Figure 2 shows a landscape of reality in which a rainbow stands out, which in terms of cutting a certain subject is a system connected to the rain and the sun, but alien to the forest object. Most likely, in the experience of such a subject, there should be records of other rainbows without the presence of forests. However, subjects who have never left their villages located in the middle of forests would not necessarily leave them out of the records. This fact suggests that the collective experience, such as that of interdisciplinary teams, adds to the possibilities of recognizing the essential connections of those experiences that are merely contingent. It should be noted that this property of consciousness that disconnects or connects perceptions is a key aspect in research via modeling.

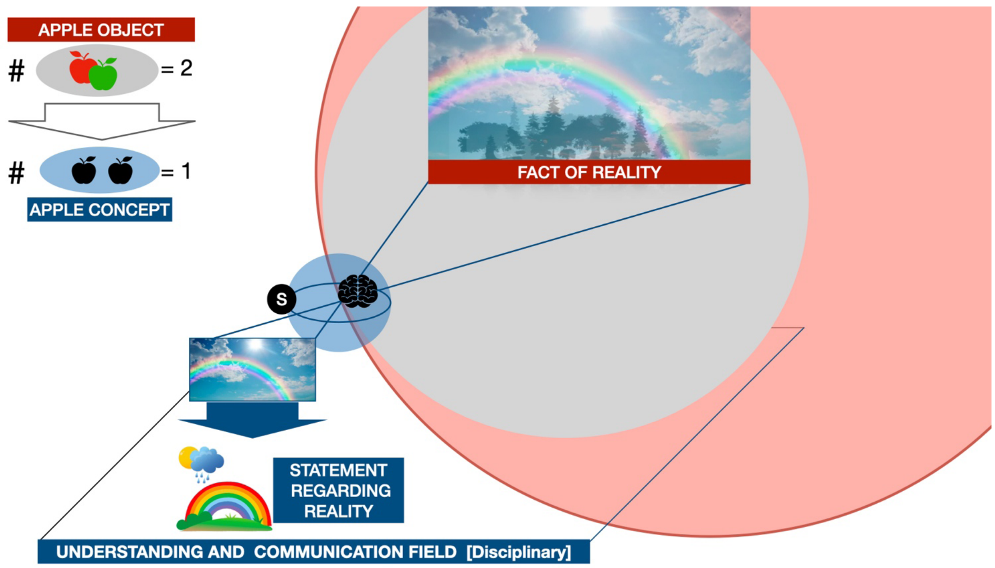

Figure 2. A disciplinary community registers and projects its experience related to specific features of common interest of the facts in the field of intellection and communication via concepts and statements. The concept should be unique, there are not two apple concepts, but pairs of apples. The statements connect concepts and are judged true (or false) by experimentation.

Given a fact, there will be as many perspectives as observing subjects. However, if they form a culture, because they have a regular investigative zeal and a similar ontological intention, i.e., they share the set of characteristics of the objects on which they focus and make use of consensual extensions of the senses (same scissors), they will be giving body to a discipline of context. These scientific communities record common experiences and the projections that they determine (what is represented), what in Figure 2 we labeled as the field of intellection and disciplinary communication. Then, the same fact-in terms of connection (or system) of objects of reality-can present as many variants in terms of stimulus object and perceived object as disciplines are interested in the fact in question.

The disciplinary field is the place where each new scientific concept is born and located, a way of labeling a family of perceived objects that have a pattern recognized by collective experience. For the logical consistency of the work with concepts, they should be unique (exhibit consensus). It frequently happens that the word associated with the concept coincides with the word used for any of its represented factors. In the upper left corner of Figure 2, we represent this fact, since two apples can exist as objects of reality, but never two apple concepts. This is not a dilemma for mathematicians, since when asked about the cardinality of the set {1/2; 0.5}, they do not hesitate to answer that it is one, a singleton, since objects are always only concepts in mathematics. Only if the observers were focused on the spelling, i.e., on the signifier, a way that is outside the mathematical intention, would he reasonably say that that cardinality is two.

We have reached a point where it is necessary to distinguish between the fields of intellection involved in interdisciplinary work. There are the context fields, being those in which their concepts are directly linked to the factual objects of interest of the disciplines. In these fields (or projective spaces) the fundamental unit of knowledge and communication is the statement, an affirmation that connects concepts and that is established with the intention of justifying why an event occurs or not. On the other hand, there is the mathematical field, in which the concepts are released from their connection with the objects of reality and begin to have an interest in themselves, in the concepts and the consistent relationships between them. This way, they become objects of study, objects of a different kind, void of concrete reality, technically called ideal objects.

In this field, we also have statements. In order to distinguish them, we prefer to use the term propositions. The two types of fields, the disciplinary and the mathematical, involve language; however, we can resort to the well-known semiotic triangle to point out differences between statements and propositions that modelers should take into permanent consideration. As the association via colors in Figure 3 indicates, the disciplinary field is superlatively oriented to the relationship of its signs with their referents, to the semantic factor, which borders on the shifting terrain of truth. On the other hand, in the mathematical space, the orientation of the tasks manages the consistency and logical derivations of the propositions among themselves, i.e., a syntactic perspective predominates. In this sense, it is possible to establish a contrasting parallelism between statements and propositions. While the former are conceptual objects with external semantics, i.e., with reference to facts, the latter refer to ideal objects with internal semantics and in which the correspondence is sought through reason, what we will refer to as thought correction. Thus, the ultimate task of modelers in interdisciplinary teams is to obtain, from the model, propositions derived by correct arguments, which, when interpreted and in the form of statements, will not only be valid, but also aspire to be true.

EPISTEMOLOGICAL DIFFERENTIATION

Regarding the millenary dispute of whether reason or experience are the genesis of our knowledge regarding the world, modelers in interdisciplinary contexts will have to build some clarity for the purposes of the coexistence that they will have to maintain between the methods of their discipline and the classic ones of the factual sciences. For example, in the hypothetical-deductive method that assumes the flow in large blocks: observation; hypothesis; and testable consequences, where is the contribution of a mathematical modeling located? Having some light is a need that will arise in the interdisciplinary interaction, which will probably be present when defining the roles and evaluating the scope of the formalist contribution.

In the factual sciences, the researchers are interested in consolidating or innovating in answers to certain research questions and, based on the analysis of previous observations, raising hypotheses, i.e., conjectures about context-causes-effects that explain the phenomena of interest. The hypotheses are normally tested through the implementation of specific experimental designs. What does a model do before a hypothesis? Can a mathematical model reject or accept a hypothesis? Do derived mathematical propositions clearly have, by interpretation, real reports as statements about reality?

With respect to the contrasts, we read: "In the hypothetical-deductive method, the contrasts should be experimental. In addition, they should be robust, in the sense that they should tell us clearly whether the consequences deduced from the hypothesis occur in reality or not" (*Sobre un concepto histórico de ciencia* [On a historical concept of science], Carlos Pérez).

As already observed, the theoretical projective field (the disciplinary) and the mathematical field concur in the work of interdisciplinary teams. It is worth noting that a scientific hypothesis is a statement of the context field. This way, it is not a fact and, in principle, neither is it a mathematical proposition; it is only linked to the facts by reference. Therefore, entering the field of truth, we observe that it is in this disciplinary field where the statements can be true or false, since in the reality field, these qualifiers do not make sense, because the facts occur or not.

Now, regarding the field of mathematics, it is better to consider other terms, since for group communication it is prudent to refer to propositions, highlighting those that have a demonstration, to distinguish between correct or incorrect deductive thought. Thus, what is desired is to reach hypothetical statements that have a mathematical correlate as correct propositions in the passage from the antecedent to the consequent and, furthermore, when interpreted, they pass the empirical tests or contrasts in order to become true. However, the latter is based on resisting a permanent test of non-falsity in the experimental field.

	Proposiciones Matemáticas Math*ematical Propositions*	
	Correctas *Correct*	**Incorrectas** *Incorrect*
Enunciados **V** *Statements* ***T***	SÍ *YES*	NO
Disciplinares **F** *Disciplinaries*	NO	NO

It is worth noting that a hypothesis is a reality bet. It is that statement with some degree of foundation and, furthermore, testable. Presupposing certain facts, it affirms that another fact necessarily follows, generally related to the specification for a perceived object of a state or a type of state within the feasible ones. Therefore, although the theoretical projective field tells us about empirical contrast through experimentation, there is also the rationality contrast, i.e., where a first opportunity for mathematical modeling arises, testing whether a hypothesis is logically consistent.

Another possibility is modeling the empirical tests (i.e., not possible to be implemented), as mental experiments, checking that their analytical conclusions go (they admit the semantic interpretation) in the direction of what the hypothesis affirms. However, a statement like a hypothesis, with its generality, even being logically consistent, can become a less plausible statement, one is never sure whether an experiment that contradicts it will appear. Thus, since the truth of a consistent hypothesis risks its plausibility in empirical control, lowering the expectations, the models only add or subtract arguments or support for their formulation.

Mono-disciplinary scientific teams usually structure or guide their work plan through a research question, and the same fact occurs in interdisciplinary teams. What type of model is appropriate for this question? What are the legitimate transfers from what is represented to the representative model (abstraction), and *vice-versa* (interpretation)? What are the minimum, first and necessary elements to consider, in order to build a model with some analogical claims regarding phenomena of reality with possibilities of contributing with an answer to the research question? It is worth being clear about the ontological distinction between objects and facts of reality, language as an articulator of concepts through propositions, and the set of ideal objects and formal relationships, the mathematical propositions that aspire to become a model.

CONCLUSION

When modeling, we should not only be aware of the differences between context and mathematics regarding objects, declarative propositions, and methods involved. There are many other aspects with philosophical traces that modelers can explore for a better understanding of their work. An enormous space tempts the exploratory desire; it is the sense of value that involves the tasks, be it of use and, why not, also the aesthetic one; however, this editorial has already gone on too long.

I end by pointing out that, in the face of a research question, its associated hypothesis and the level of precision of the answer, apart from what has already been stated, it is useful and beautiful that the modelers aspire to constructions that perform the task in the simplest possible way. For the present and other topics, it is worth recommending the book *"Deja a la estructura hablar: Modelización y análisis de sistemas naturales, sociales y socioecológicos"* (Let the structure speak: Modeling and analysis of natural, social and socioecological systems) by Dr. Rodrigo Ramos.

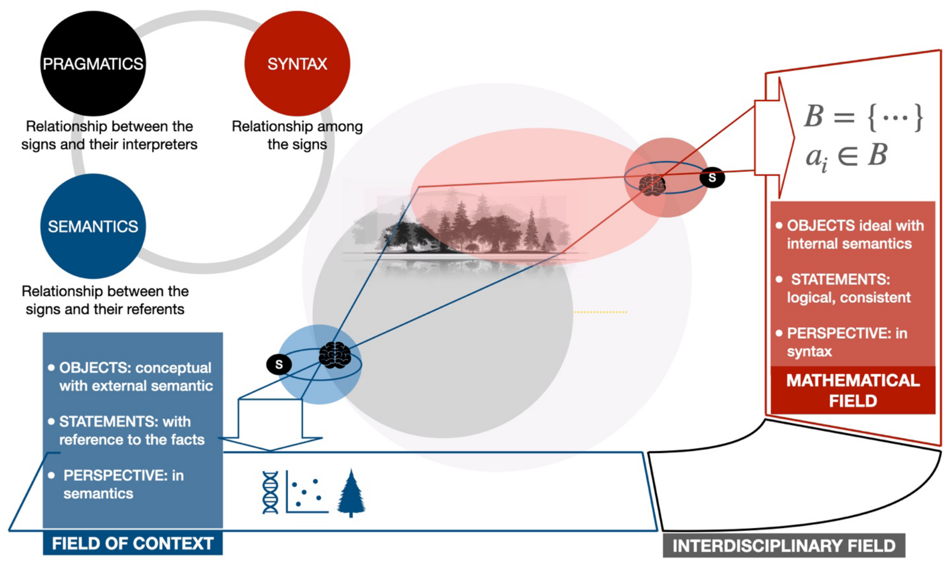

Figure 3. At the field of context, the statement has significance in the facts. Its semantics is external; it occurs or not in its real referents. On the other hand, in mathematical propositions, the referents are only concepts, ideal objects, internal semantics; syntax and the relationship among the signs are highlighted. The interaction defines, by adaptation, the interdisciplinary field of communication in which statements of context and mathematical propositions coexist. Regarding pragmatics, the third semiotic vertex, it is the concern of other imaginable fields, e. g., those related to psychology or educational sciences

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REVISTA MMSB

Mathematical model as a management tool to analyze organic matter self-purification in reservoirs

Modelo matemático como herramienta de gestión para analizar la autodepuración de materia orgánica en embalses

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Abstract—A mathematical model to analyze the self-purification potential of organic matter in reservoirs was developed, seeking to constitute a potential water management tool for easy application. Therefore, it was considered the input of organic matter into the watercourse, its sedimentation, decomposition and its output downstream, as well as the oxygen input through the flow and through re-oxygenation, its consumption by microbial activity and its downstream exit. The model was validated using data of Tietê river and applied using data of Salto Grande reservoir, both located at São Paulo State, Brazil, considering different scenarios of biochemical oxygen demand (BOD), dissolved oxygen (DO), water inlet flow and reservoir water volume. The results show that organic matter is decomposed faster in the first 12 hours of water travel; moreover, self-purification efficiency is better with a greater volume of water in the reservoir. The model is quite representative to study different self-purification scenarios in dammed areas, concluding that the developed model, being tested and improved, can already contribute to the management of reservoirs, with the advantage of having a simple application.

Keywords—Biomathematics, Dams, Superficial water quality, Water contamination, Water monitoring, Organic carbon mineralization

Resumen—Se desarrolló un modelo matemático para analizar el potencial de autodepuración de la materia orgánica en embalses, buscando constituir una potencial herramienta de gestión del agua de fácil aplicación. Para eso, se consideró el ingreso de materia orgánica al curso de agua, su sedimentación, descomposición y su salida aguas abajo, así como el aporte de oxígeno por flujo y por reoxigenación, su consumo por actividad microbiana y su salida aguas abajo. El modelo fue validado con datos del río Tietê y aplicado con datos del embalse de Salto Grande, ambos ubicados en el Estado de São Paulo, Brasil, considerando diferentes escenarios de demanda bioquímica de oxígeno (DBO), oxígeno disuelto (OD), flujo de entrada de agua y volumen de agua del embalse. Los resultados muestran que la materia orgánica se descompone más rápido en las primeras 12 horas de viaje por agua; además, la eficiencia de la autodepuración es mejor con un mayor volumen de agua en el depósito. El modelo es bastante representativo para estudiar diferentes escenarios de autodepuración en áreas embalsadas, concluyendo que el modelo desarrollado, todavia en prueba y mejorías, ya puede contribuir a la gestión de embalses, con la ventaja de tener una aplicación sencilla.

Palabras clave—Biomatemática, Represas, Calidad de agua superficial, Contaminación de agua, Monitoreo de agua, Mineralización de carbono orgánico

INTRODUCTION

Due to the misuse of superficial water, many tributaries of reservoirs end up carrying (and storing in their sediments) contamination loads from the lack of domestic and industrial sewage treatment. Also, they have their flow rates altered due to innumerable anthropogenic interference in river courses (Curbani et al., 2021; Bianchini Junior, 1999; Bianchini Junior and da Cunha-Santino, 2018)

The concentration of contaminant compounds, however, can naturally be reduced by water bodies through the self-purification process. It is the natural ability of an aquatic ecosystem to depurate organic and inorganic compounds through biological, biochemical and physical-chemical processes. Decomposition of compounds by aquatic microorganisms, chemical oxidation of organic matter, and inlet and outlet of gases in surface waters are the main factors that characterize the water self-purification process (de Esteves, 2011).

It is important to understand that self-purification transforms excess organic compounds (which is harmful to the aquatic ecosystem) into inert compounds that are no longer harmful to organisms. This purification does not necessarily bring potability to water, but restores the original characteristics of a water body that has suffered some kind of contamination, returning the equilibrium to the ecosystem (Suslova et al., 2018).

For conceptual purposes, self-purification process can be divided into zones: 1) the degradation zone, where effluent is discharged; 2) active decomposition zone, where dissolved oxygen reaches its lowest concentration and organic matter is mostly decomposed; 3) recovery zone, in which the dissolved oxygen begins to be reintroduced, algae and some species of fish reappear; and 4) clean water zone, where dissolved oxygen is recomposed in the environment and the excess organic material has already been degraded. Also, the presence of species of algae, fungi and bacteria and nutrient concentration are characteristic of balanced environments (Dodds and Whiles, 2020).

The type of watercourse influences the extent and time of each of these zones. In a lentic system with less water movement, purification is more dependent on water residence time; whereas in the lotic system, like a river, with almost continuous movement, purification is related to the space covered by water (Wetzel, 2001). Each water body has its purifying potential, which depends on ecosystem characteristics such as temperature, salinity, biodiversity, length and depth, water velocity and flow. This potential can be affected by climatic variations such as annual temperature, wind and humidity (Delgadillo et al., 2010). Also, the characteristics of wastewater discharged into water sources affect the self-purification potential (Skulovich et al. 2018). The composition of organic material, the presence of inhibitory substances and the disposal of (Skulovich and Ostfeld, 2018).Self-purification capability of a stream has to be known by the monitoring organs. They determine in what quality stage each watercourse belongs to and for what purpose it can be used, among all of the possible uses of a watercourse. This capability is also a necessary information for those who develop and maintain sewage treatment stations. In this sense, the application of mathematical modeling can represent to the state monitoring agencies a viable and low-cost alternative for management understandings and subsidies, favoring the administration of water volume and flow to the downstream river.

Due to the problems related to reservoir constructions (Federice, 2014), developing and applying mathematical models in dammed areas can be strategic in order to: scale the damage caused by damming; impacts caused by the damming ecosystem; changes in river flow downstream of the reservoir construction; growth and death of organisms; purification of organic material; increased area of gas exchange between air and atmosphere; effects of thermocline created by incorporating lacustrine environment into a previously fluvial environment; physical effects of diffusion of water; among other analysis that can be performed for many purposes, using mathematical modeling as a tool.

Several self-decay studies have been performed to evaluate the potential for purification in a watercourse (Blodau et al., 2018; de da Cunha et al., 2018; Ricciardone et al., 2011; LaRowe and Van Cappellen, 2011; de Silva et al., 2007), to analyze the impact of reservoir construction in water self-decay capability (Wei et al., 2009), to evaluate the clearance of organic matter in marine sediment (Arndt et al., 2013), to understand the impact of some effluents in the self-purification capability (Mengistie et al 2016), to analyze the poly-functional role of biota in water self-decay capability (Ostroumov 2017), to evaluate the decay potential in a constructed wetland (Ophithakorn et al., 2013) and to analyze, in addition to self-purification capacity, the deoxygenation and reoxygenation rates of the environment (de Menezes et al., 2015), among others.

Regarding models performed to study reservoir water quality, CE-QUAL-R1 was developed for temperature, chemical species and biological assemblies (Wlosinski and Collins, 1985);the model CE-QUAL-RI is a complement to the previous model and includes vertical oxygen concentration, some algae groups, particulate organic matter and dissolved organic matter, among other parameters (MCA Filho et al., 1990),and CE-QUAL-W2 (Cole and Wells, 2006),that is a two-dimensional model. CE-QUAL-RI and CE-QUAL-W2 models are quite complete, but the high number of parameters contained in them restrict their application; Mike models evaluate hydrodynamics processes (MIKE, 2007). For both models, the user needs to purchase a license; (Vázquez and Mokrova, 2019) present the functionalities integrating mathematical models to the Geographic Information System (GIS) for studies of only hydrological and hydraulic processes; (Jiang et al., 2016) developed a model to assess the effect of reservoir regulation on water quality; and (Reartes et al., 2016) developed a model including as parameters the main taxonomic groups of phytoplankton in reservoirs, some nutrients, dissolved oxygen, biochemical oxygen demand and cyanobacteria. However, it was developed to be used only in stratified reservoirs and therefore not in all of the reservoirs. All of these models have the common property of not having such an intuitive implementation and are not focused on the analysis of water self-purification.

In this work, it was developed a mathematical model to evaluate the self-purification capacity in water reservoirs and other flooded areas with intermediate system profiles. It has the advantage to be composed by simple parameters and is easy to be implemented, assuming as hypothesis that this is an innovative model compared to the works presented in the literature. Thinking the mathematical modeling as an object representation tool, in order to study the interaction between its variables under different situations, a system of mathematical equations was developed. These equations represent the behavior of organic material and dissolved oxygen in a reservoir since its input with upstream river water flow, to its output through the dam, and has the purpose to be a simple and easy model to apply.

THE MODEL

For the composition of this model, it was considered the purification process of organic matter present in the reservoir: by the dilution of compounds that occurs in the transition space between lotic (riverine region, that is, beginning of the reservoir) and lentic (damming water near barrage) zones; by the biodegradation, which is part of the trophic chain existing in both environments; and by the sedimentation that occurred mainly in its lentic region. All of these adopted interactions are performed mainly in the epilymium of the dammed body.

Thus, the developed model proposes to evaluate the amount of oxygen O_2 and carbon C, both as a function of continuous time t, with $t \in [0,T) \subset R$, where $\frac{dO_2}{dt}$ and $\frac{dC}{dt}$ represent the rates of variation of this amount of oxygen and carbon as a function of time, in the case of reservoirs with a downstream river outlet.

Figure 1 presents a sectional drawing of the dam and the interactions considered in the composition of the model.

In Figure 1, DIC represents dissolved inorganic carbon, DOC represents dissolved organic carbon and POC, particulate organic carbon. The interactions among contributions of effluents, macrophytes, phytoplankton, DOC and POC with heterotrophic organisms represent the mineralization and metabolic consumption of organic carbon. The interactions between DIC and CO_2 represent the gas exchange between air and water.

Table 1 shows what each of the interactions in Figure 1 represents in the developed model:

TABLE 1: INTERACTIONS CONSIDERED IN THE RESERVOIR AND THEIR RESPECTIVE REPRESENTATIONS IN THE DEVELOPED MATHEMATICAL MODEL

Reservoir	Model
Contributions of effluents and macrophytes	Input of carbon: $\frac{Q_i}{V}C_0$
Water flow of the river(s) upstream	Inlet flow: $\frac{Q_i}{V}$ Input of oxygen: $\frac{Q_i}{V}C_0$
Gas exchange and air-water interactions	Reoxygenation constant: k_r Input of oxygen: $k_r[O_{sat} - O_2]$
Mineralization and metabolic consumption of C	Deoxygenation constant: k_d Proportionality between C and O_2: β
Dissolved oxygen	Oxygen consumption and production in epilymium: $k_r[O_{sat} - O_2]$ and $\beta k_d C$ Reoxygenation constant: k_r
Sediment-water interactions	Non-linearity between load and sedimentation: γ Sedimentation constant: μ
Downstream water discharges	Outlet flow: $\frac{Q_{s_1}}{V}$

For the oxygen variation rate as a function of time, Equation (1), we considered: " *the reoxygenated oxygen portion, plus the portion of oxygen that enters the epilymium, minus the portions of oxygen that flows out to the downstream river, less a portion of carbon that deoxygenates*".

Mathematically, this is:

$$\frac{dO_2}{dt} = \frac{Q_i}{V}O_{2_0} + k_r[O_{sat} - O_2] - \beta k_d C - \frac{Q_{s_1}}{V}O_2 \qquad (1)$$

Where

$$C = \frac{BOD}{\beta} \qquad (2)$$

Regarding the carbon variation rate as a function of time, Equation (3), we considered: " *the portion of carbon that enters the epilymium, minus the portion of carbon that decarboxylates, minus the portion of carbon that sediment, minus what goes out with the flow downstream*".

Mathematically, this is:

$$\frac{dC}{dt} = \frac{Q_i}{V}C_0 - k_d C - \mu C^{\gamma} - \frac{Q_{s_1}}{V}C \qquad (3)$$

With $O_2(0) = O_{2_0}$ and $C(0) = C_0$, where:

- $O_2 = O_2(t)$ indicates the oxygen of the epilymium (mg/m^3);
- Q_i indicates the inlet flow (m^3/d);
- V represents the water volume of the reservoir (m^3);
- O_{2_0} indicates the input of oxygen (mg/m^3);
- k_r represents the reoxygenation constant (d^{-1});
- O_{sat} represents the saturation oxygen in the epilymium (mg/m^3);
- β indicates the proportionality constant between C and O_2 (d^{-1});
- k_d indicates the deoxygenation constant in the epilymium (d^{-1});
- Q_{s_1} indicates the outlet flow (mg/m^3);
- $C = C(t)$ represents the carbon in the epilymium (mg/m^3);
- BOD represents the biochemical oxygen demand (mg/m^3);
- C_0 represents the input of carbon (mg/m^3);
- μ indicates the sedimentation constant (d^{-1});
- γ represents the non-linearity between load and sedimentation.

The model was solved numerically using one 4^{th} order Runge-Kutta Method (RK4), widely used to solve initial value problems, given its simplicity of implementation and high precision (Carnahan and Wilkes, 1973; Cunha, 2001), with local truncation error on the order of $O(h^5)$. To implement the problem, Matlab®was used.

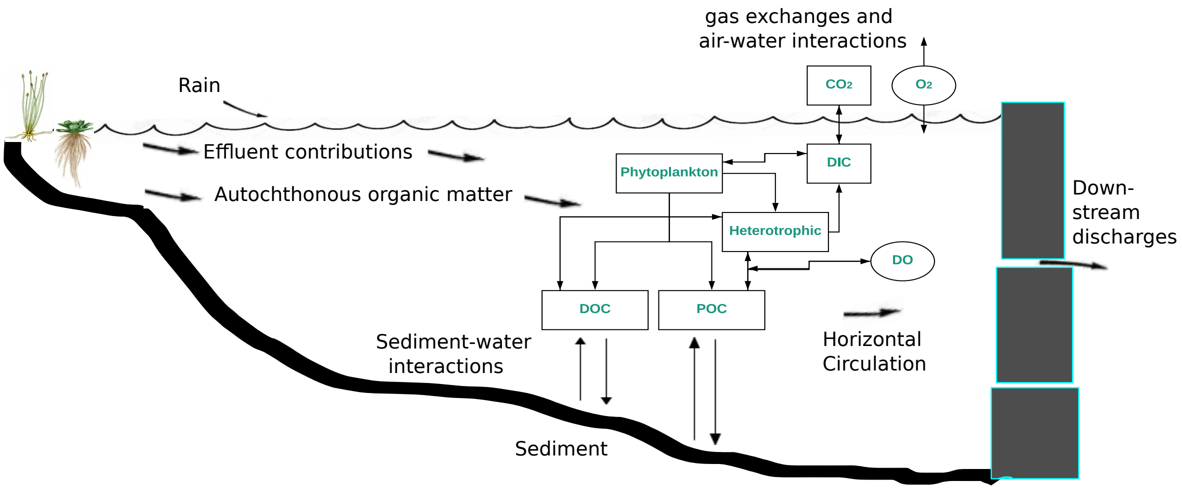

Figure 1: Interactions considered in the reservoir to develop the model. Source: adapted from (de Esteves, 2011) and (Tundisi, 2013).

Model Validation

The validation of the developed model (Equations 1 and 3) was performed using data from collections by Tercini and Méllo Junior (2016) and Tercini (2014) The authors evaluated the self-purification of the Tietê River from the municipality of Pirapora to Salto, in São Paulo State, Brazil, through an adaptation of the Streeter & Phelps model. The Tietê river stretch considered has small reservoirs. The field work carried out by the authors consisted of monthly water sample collections and analysis, in 2012, of biochemical oxygen demand (BOD), dissolved oxygen (DO) and temperature in some points of the river and reservoirs. They also stipulated the reaeration and deoxygenation constants for the lentic and lotic stretches.

Considering the model proposed in this study, the self-purification of the Tietê River stretch is evaluated, adopting the input data from BOD (which represents the input of carbon) and DO downstream and upstream of the Rasgão reservoir studied by Tercini (2014). Thus, we were able to analyze whether what was depurated at the outlet of the reservoir coincides with the result predicted by the model.

Figure 2 presents Tietê sub basin chosen by Tercini and Méllo Junior (2016) to perform the study:

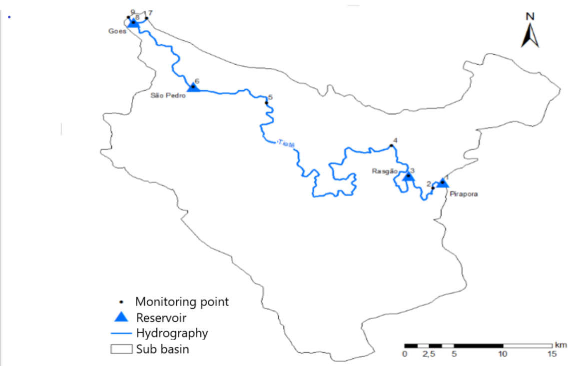

Figure 2: All the collections points chosen by Tercini and Méllo Junior (2016) through the Tietê sub basin between Pirapora and Salto cities. Source: Tercini (2014).

Figure 3 shows the collection points performed by Tercini and Méllo Junior (2016). selected to validate the developed model (Equations 1 and 3) (which we call Points 1, 2 and 3):

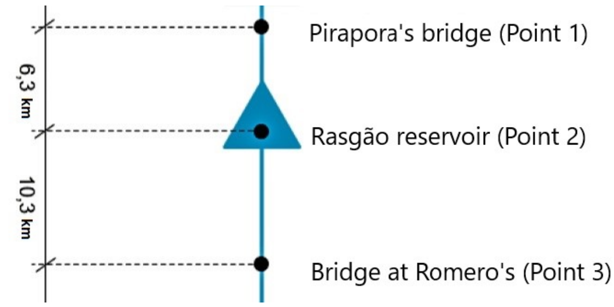

Figure 3: Point 1, Point 2 and Point 3 of the collections carried out by Tercini and Méllo Junior (2016). in the Tietê river that were used to validate the developed model. Source: adapted from Tercini (2014).

Point 1 (Pirapora's bridge) provides input data for the validation of the developed model. Considering 6.3 km of river stretch until the arrival at the reservoir at Point 2 (Rasgão Reservoir), the self-purification of this river stretch was determined using the Streeter & Phelps model, which is more appropriate for river sections. Equation 4 presents the model:

$$\frac{dO}{dt} = -k_d L + k_r (O_{sat} - O_2) \tag{4}$$

Thus, for the simulations in this section, we applied the Streeter & Phelps model and used the values of BOD, OD, k_d, k_r, average velocity and distance between sections shown by Tercini et al. (2016), according to Table 2. The BOD and DO results used were from May 2012 analysis accomplished by Tercini et al. (2016).

Figure 4 expresses the simulation between Point 1 and Point 2 using the Streeter & Phelps model:

The final BOD after 6.3 km of the route, or 5.1h, was $2.786\text{x}10^4$ mg/m^3 and the final DO was $3.907\text{x}10^3$ mg/m ^{3}. These results were used as initial BOD and DO to simulate the next section, which is the reservoir.

In the reservoir, the developed model was applied using the parameters: k_d of 0.183/day, k_r of 0.009/day, inflow of 11230000 m^3 /day, outflow flow of 10329984 m^3/day, reservoir volume of 6677000 m^3 and water residence time in the reservoir of 15.51h (0.65day) (Tercini et al. 2016). We

Table 2: Values of BOD and OD from May 2012, k_d, k_r and average velocity extracted from Tercini and Méllo Junior (2016). for model validation. Source: Tercini and Méllo Junior (2016)

Parameters	Value
BOD	2.9×10^4 mg/m^3
DO	4.15×10^3 mg/m^3
k_d	0.188/d
k_r	1.12/d
Average velocity	29,635.2 m/d

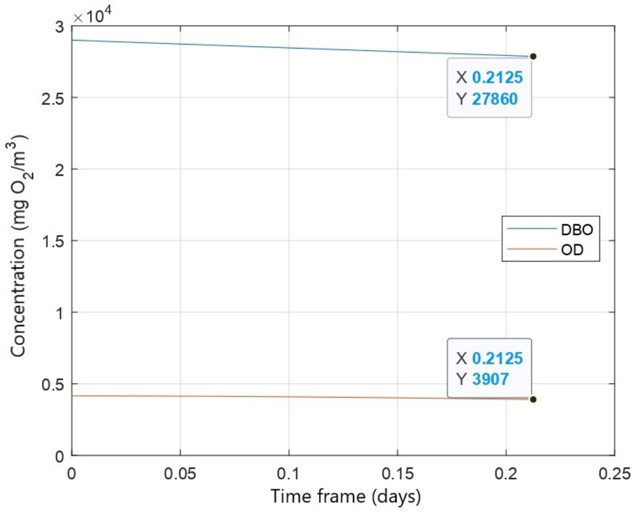

Figure 4: Simulation of the behavior of BOD and DO between Points 1 and 2 of the validation, that is, in the first section.

adopted, following (Margalef, 1983), proportionality constant between C and O_2 (β) of 2.6/day and nonlinear relation between load and sedimentation (γ) of 0.8/day, and sedimentation constant (μ) of 1.0/day (tabulated by (Mizael, 2019)). The generated simulation is shown in Figure 5:

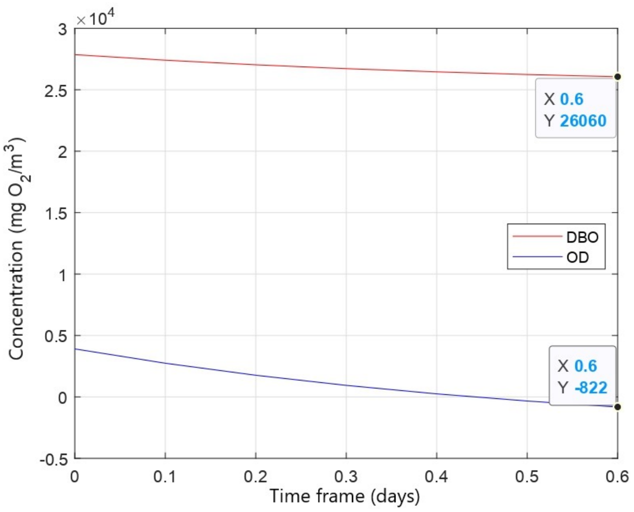

Figure 5: Simulation of the behavior of BOD and DO in the Rasgão reservoir, that is, in the second section.

The endpoint of the simulated curve for BOD is 2.606×10^4 mg/m^3 and for DO is -8.22×10^{-1} mg/m^3. It is noteworthy that, as it is not possible for a negative oxygen concentration to exist, this negative moment of the oxygen curve is attributed to anaerobiosis, that is, to the decomposition carried out by anaerobic microorganisms. Considering the next collection point by Tercini and Méllo Junior (2016)(Point 3) at 10.3 km after the reservoir (or 0.35 days of travel), the Streeter & Phelps model was again applied between Point 2 and Point 3 (bridge on the Romeiros road) to estimate the organic decomposition in the stretch of the river.

Using the end points of the curves in Figure 5 as the initial condition for the simulation, the constants used were the same applied in the simulation of the section between Point 2 and Point 3 (k_d of 0.188/d k_r1.12/d). The simulation is shown in Figure 6:

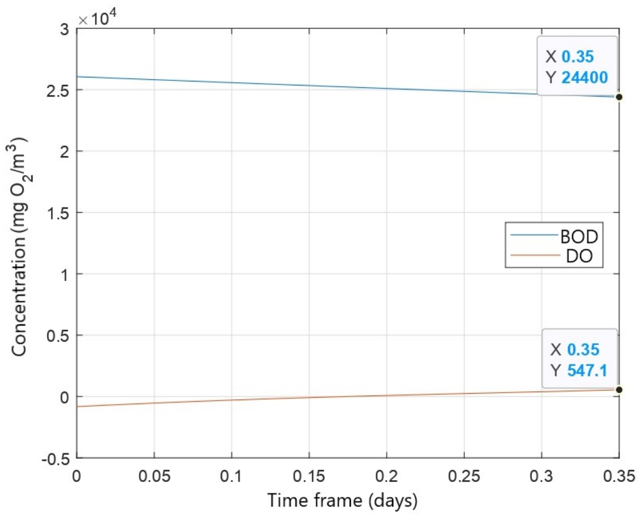

Figure 6: Simulation of the behavior of BOD and DO between the output of the Rasgão reservoir and Point 3, that is, in the third section.

The analysis by Tercini et al. (2016) showed, for Point 3, BOD of 25000 mg/m^3 and DO of 480 mg/m^3. The values obtained using the mathematical modeling were: BOD of 24400 mg/m^3 and DO of 547 mg/m^3. The difference between the empirical results found by the authors and those predicted by the proposed model are 600 mg/m^3 for BOD and 67 mg/m^3 for DO. This is qualitatively extremely similar, considering the dynamism of aquatic environments and the margins of error applied to empirical analysis of Physical-Chemical parameters (APHA 2017). Thus, it is concluded that the model is valid for the study of self-purification in reservoirs.

Salto Grande Reservoir, Americana, São Paulo, Brazil.

The model developed was applied in a case study performed in Salto Grande Reservoir, located at the city of Americana, 120 km far from São Paulo city, Brazil. This reservoir is at a populated macro zone, with around 3 million people and very important economically to the country due to its industrial and agricultural development (IBGE 2019).

The dam was constructed in the 40's and started to work in 1950, producing electrical energy in its small hydroelectric plant (Bottura, 1998). The dammed river for its construction is called Atibaia and it is an important river for São Paulo State, providing water for Cantareira System and supplying large cities like Campinas, with almost 1.2 million inhabitants (Montagner and Jardim, 2011).

Salto Grande Reservoir has a contribution area of 2724 km^2 and residence time from 10 days (summer) to 4 months (winter) (Mizael, 2019), the reservoir is eutrophic, since Atibaia river receives disposal of liquid effluents from many of the cities it crosses, taking to the dam large concentrations of organic matter, nutrients and several compounds that lower the quality of the water body (Rodrigues et al., 2019).

Surface water collection and BOD and DO analysis of Salto Grande reservoir samples were performed in five different points along the reservoir (P1, P2, P3, P4 and P5) and one point downstream (P6) with the intention to pursue a self-decay evaluation using the model presented at this study, that could after be applied at any reservoir. The distance between the points within the reservoir varies from 2 to 2.7km. The distance between the dam and P6 is 600m, since P6 collection happened from a bridge that is the closest access to water downstream from the dam. This spacial variation is not considered within the model. The collection points can be seen in Figure 7.

Each collection point presents coordinates and some characteristics are exposed in Table 3:

TABLE 3: CHARACTERISTICS OF EACH OF THE COLLECTION POINTS.

Point	Coordinates	Depth (m)[1]
P1	22^{o}43′47,68”*S* and 47^{o}13′18,44”*O*	
Characteristics	Entry point of the Atibaia river; presence of aquatic plants; some crops on the nearby banks.	2.62
P2	22^{o}43′18,10”*S* and 47^{o}14′5,24”*O*	
Characteristics	Reduced distance between margins; presence of algae (in most of the time).	11.59
P3	22^{o}42′58,58”*S* and 47^{o}15′5,94”*O*	
Characteristics	Area with small farms on the banks.	8.56
P4	22^{o}42′57,05”*S* and 47^{o}16′5,85”*O*	
Characteristics	Small farms on one of the banks; native vegetation and crops on the other bank.	9.95
P5	22^{o}42′2,43”*S* and 47^{o}16′18,13”*O*	
Characteristics	Presence of aquatic plants; small farms on one of the banks; small native vegetation area and crops on the other bank.	13.79
P6	22^{o}41′46,02”*S* and 47^{o}17′20,73”*O*	
Characteristics	River point; water with little turbidity and high velocity	-

The samples were collected and stored in glass bottles under refrigeration in a polystyrene box. All analysis were performed in the Physical-Chemical Laboratory of the Faculty of Technology, State University of Campinas, with certified technical assistants and following the standard methods for each of the analysis (APHA, 2017), within 2 hours after collection.

The result of the first water collection point was used as the BOD and DO intake at one simulation with the model. All of the other points at this simulation, each of them 2 days far from the others in summer season, were compared to the empirical results of Salto Grande reservoir.

Besides, some analysis were accomplished considering hypothetical scenarios in the reservoir. The main objective of this is evaluating changes in the purification disposition of organic matter under the occurrence of some phenomena, such as effect of effluent disposal, changes in inlet flow and volume. Thus, the following hypothetical cases were evaluated:

- Case 1: there happens the occurrence of average income of organic material and average volume of the reservoir, so a normalized average of BOD concentration input, inlet flow and volume of the reservoir was also maintained;
- Case 2: there happens the occurrence of high inflow of organic material from the Atibaia River, with normalized average of inlet flow and volume of the reservoir, that is, organic matter intake was increased and dissolved oxygen was reduced;
- Case 3: there happens the occurrence of average organic matter concentration, with high inlet flow and average volume of the reservoir, so there is a substantial increase in the flow of the Atibaia River and, therefore, an inflow increase;
- Case 4: there happens the occurrence of high inflow of organic material from the Atibaia River, with low water inlet flow and average volume of the reservoir;
- Case 5: it was compared the self-purification process in the reservoir from two perspectives: the first (a) with low water volume, average inlet flow and high organic matter inlet flow concentration and the second (b) with low water volume, average inflow and low organic matter inflow concentration.

All the analysis were carried out considering 10 days of total water travel time inside the reservoir. Table **??** shows the values used in the simulations.

[1]The average depth of each point was calculated based on the depth analysis performed in Cunha (2020)

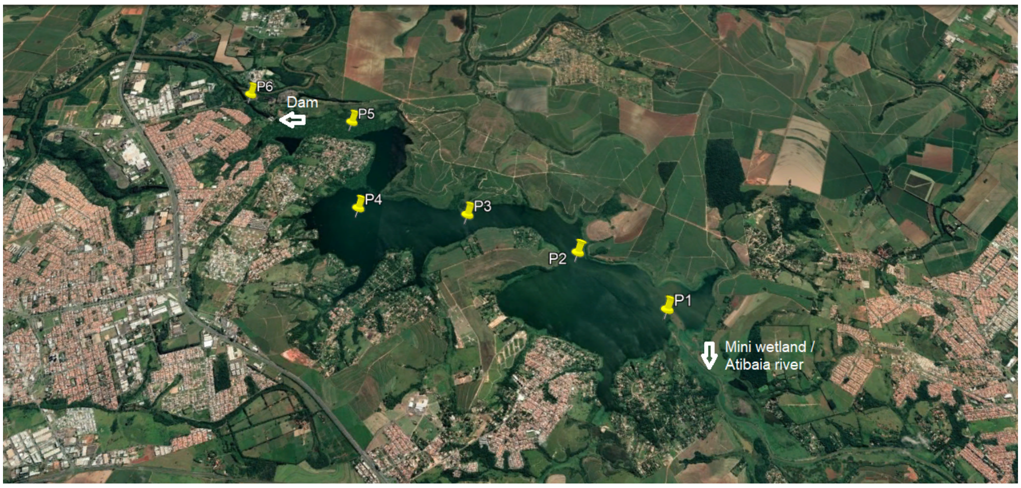

Figure 7: Water collection points from Salto Grande reservoir for BOD and DO analysis. Source: adapted from Google Earth®.

	Input flow (m^3/s)	Volume (m^3)	BOD (mg/m^3)	DO (mg/m^3)	Output flow (m^3/s)
Case 1	60.2	1.06×10^8	12000	5000	54.2
Case 2	60.2	1.06×10^8	20000	5000	54.2
Case 3	106.5	1.06×10^8	12000	5000	95.9
Case 4	12.8	1.06×10^6	20000	5000	11.5
Case 5 (a)	60.2	1.06×10^6	20000	5000	54.2
Case 5 (b)	60.2	1.06×10^8	8000	6000	54.2

TABLE 4: FLOW, VOLUME, ORGANIC CONCENTRATION AND DISSOLVED OXYGEN USED IN THE SIMULATIONS.

The inflow of 60.2 m^3/s was assumed based on the average flow of the Atibaia River in the rainy season (DAEE, 2019). High and low flow rates were assumed. All outlet flows were assumed to be 10% less than inlet flows, as presented by (Tercini, 2014). The normal volume of the reservoir, 1.06×10^8, is presented by Espíndola et al. (2004) and the reduced volume was stipulated, as well as the concentrations of dissolved oxygen and organic matter. Volume was assumed to be constant along the reservoir. The following constant values were used in all simulations: k_r=0.12 and k_d=0.19 (according to the table presented by Von Sperling (2014), regarding constants used for reservoir ecosystems), μ=1.03 (tabulated by Mizael (2019)), γ=0.8 and β=2.6 (Margalef, 1983).

RESULTS

Salto Grande Reservoir surface water analysis and simulation

Table 5 presents the results of BOD and DO analysis of surface water samples collected in Salto Grande reservoir on April 29, 2019.

TABLE 5: RESULTS OF BOD AND DO ANALYSIS OF SURFACE WATER SAMPLES COLLECTED IN SALTO GRANDE RESERVOIR.

Date	Analysis	P1	P2	P3	P4	P5	P6
04/29/2019	BOD (mg/m^3)	5040	3460	1490	1190	1220	980
	DO (mg/m^3)	6950	6440	6310	4880	3630	6030

Figure 8 shows the simulation of the self-purification curves for BOD and DO on April 29, 2019 using the results presented in Table 5 and using the model developed.

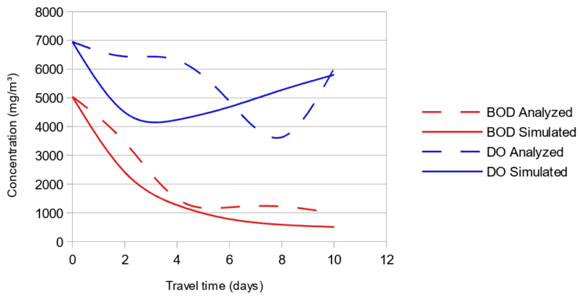

Figure 8: Simulation of the self-purification curves for BOD and DO on April 29, 2019 using the model developed.

Both BOD and DO curves start with the same concentration, since the initial concentration for the simulation curves are the concentrations analyzed in the collection whose results are in Table 5.

In the case of DO, it is observed that the simulated curve shows a notable decay in the first two days of travel. This is due to DO consumption in the system by the decomposition of organic matter, and subsequent growth from the third day of travel. However, the DO decay observed in the field assessment starts at P4 and intensifies at P5, reaching about 6000 mg/m³ at P6, a value very close to the tenth day of simulated travel.

The DO values diverge between day 0 and 10 because the collections were carried out on the same day. Therefore, the analyzed DO curve contains information that the simulations cannot give us from the path, such as: punctual changes in organic load in the reservoirs from aquatic plant banks or algae blooms; punctual changes caused by punctual contamination, such as runoff, not considered in the model; resuspension of organic matter from the deep zone.

Nevertheless, the graph is relevant for the study of the DO concentration after 10 days, presenting the same result for the final DO with the same initial DO, considering statistical error margins.

Also, for DO, the greatest variation between measured and simulated concentration was 2075 mg/m^3 at P2. The average variation between all points was 1217 mg/m^3. The empirical result of the DO concentration in the fifth point, that is the closest point to the dam that was analyzed was lower than the expected for the self-purification curve at this point. This is possibly because of the presence of macrophytes in this area of the reservoir, which reduce the gas exchange between air and water. In addition, the increase in the concentration of particulate organic matter causes a reduction in dissolved oxygen consumed by decomposing microorganisms.

In the case of BOD, both curves (analyzed and simulated BOD) evolve very similarly in relation to the travel days, ending very close to 1000 mg/m³ on the tenth travel day / P6.

The greatest variation between the measured and simulated BOD concentration was 1044 mg/m^3 at P1 and the average range of variation for all points was 556 mg/m^3, which is quite low.

Although this simulation was not performed in order to validate the model, it is possible to observe that the results achieved with this simulation are qualitatively similar to the results obtained at the same points through empirical analysis.

Case 1

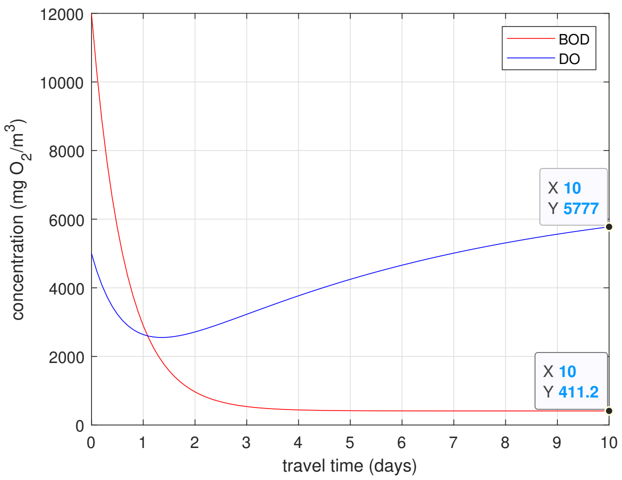

Figure 9: Case 1: normal flow, volume and organic concentration in 10 days of travel time

It is possible to observe in Figure 9 that in cases of average inlet and outlet flows, as well as the volume and concentration of organic matter and dissolved oxygen input, BOD would be reduced to less than half of its original concentration within the first 12 hours of travel. In the case of the analyzed scenario, BOD reduced from 12000 mg/m^3 to approximately 5700 mg/m^3 in the first half part of the day.

Dissolved oxygen, on the other hand, was reduced from 5000 mg/m^3 to about 3200 mg/m^3, since it is consumed by the organic material mineralization and also through oxidation.

In the 24 hours of travel, BOD decay is intensified, reaching approximately 2900 mg/m^3, and dissolved oxygen reaches 2600 mg/m^3.

It is also possible to observe that the dissolved oxygen concentration, in this analyzed scenario, would increase again on the second day of the journey, while the BOD concentration would reach stability, that is, would stop concentration decrease on the third day of the journey.

Case 2

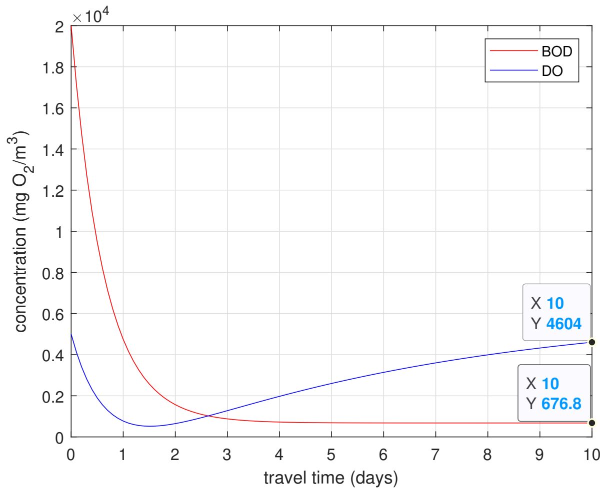

Figure 10: Case 2: Normal flow and volume and high organic concentration in 10 days of travel time

Regarding the second case analyzed, BOD is also reduced in the period of 12 hours of travel, but less than in the first case. This is attributed to the fact that the initial BOD concentration was 20000 mg/m^3. In Figure 10 it is possible to observe that BOD reaches, after half day of travel, approximately 9500 mg/m^3, while the DO reaches approximately 1900 mg/m^3. That is, it is drastically reduced by organic material decomposition. After 24 hours of travel, BOD and DO continue to decline, reaching a concentration of 4700 mg/m^3 of BOD and 770 mg/m^3 of DO, very close to anaerobiosis.

Dissolved oxygen concentration increases again on the second day of the journey, and BOD stabilizes on the third day. The reduction rates of organic matter are similar in this second case and in the first one, with the difference that in the first case, the dissolved oxygen reaches a greater value and the organic concentration a lower value in the same travel time as the second case.

Case 3

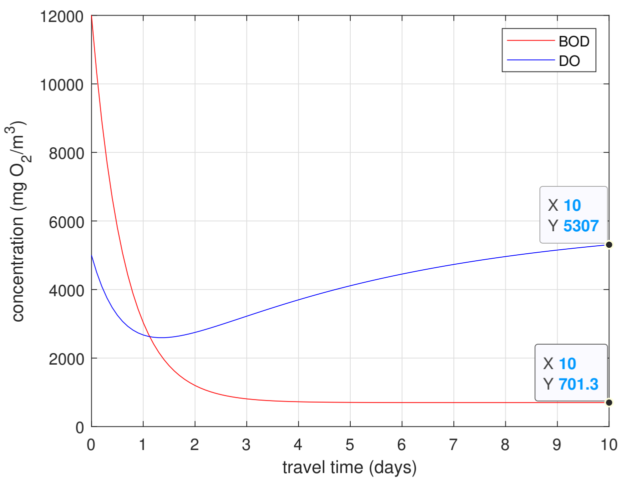

Figure 11: Case 3: High flow, normal volume and organic concentration in 10 days of travel time

Case 4

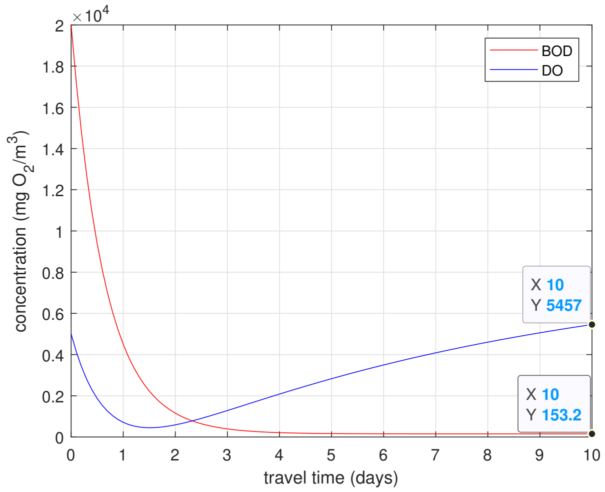

Figure 12: Case 4: low flow, normal volume and high organic concentration in 10 days of travel time

According to Figure 11, the flow increase also causes a greater inflow of organic concentration and dissolved oxygen. That is, the concentration of BOD and DO in mg/m^3 increases mathematically when there is a greater flow in m^3. So, even if BOD and DO concentrations are the same simulated in the first case (12000 mg/m^3 of BOD and 5000 mg/m^3 of DO), there are more liters of water entering the reservoir, then more BOD and DO. Therefore, it is possible to observe that BOD and DO concentration after 12 or 24 hours of travel is slightly greater than the concentrations found in the first case: BOD ranges from 12000 mg/m^3 to 5800 mg/m^3 after 12 hours and to 3070 mg/m^3 after 24 hours; dissolved oxygen, from 5000 mg/m^3 to 3200 mg/m^3 after 12 hours and to 2670 mg/m^3 after 24 hours.

It was observed that, although the oxygen concentration also starts to increase on the second day of the journey, as in the previous cases, it stops being reduced before the other analyzed cases. In other words, the inflection of the DO curve occurs before what happened in the previous cases, which demonstrates that the degradation and active decomposition zones (which are the zones of greatest oxygen consumption in the self-purification process) are smaller in this case. This is because the entry of dissolved oxygen was initially greater and, therefore, the organic decomposition is faster.

Figure 12 show that, even with low flow and high organic concentration entered in the reservoir (20000 mg/m^3 BOD), the organic matter is strongly depurated in the first 12 hours, reaching half of its initial concentration in this travel time.

Dissolved oxygen, in turn, reduces to a fifth part after 24 hours of travel, close to anaerobiosis. It is observed that it is only at the end of the second day of the journey that the DO concentration begins to increase in the reservoir, indicating that the recovery zone in the self-purification process happened later than in the previous cases analyzed .

Case 5

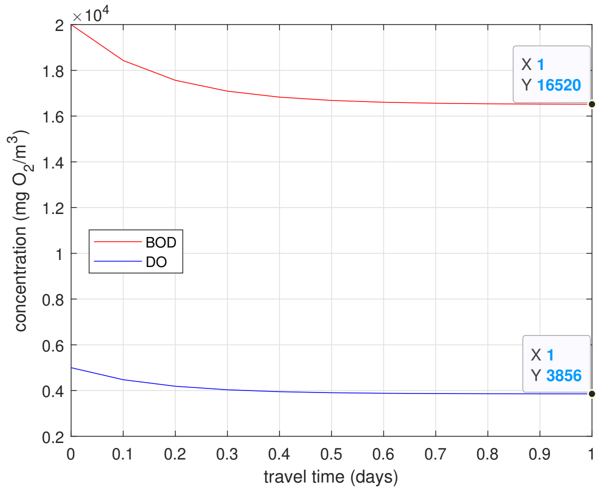

Figure 13: Case 5 (a): normal flow, low volume and high organic concentration in 24 hours of travel time

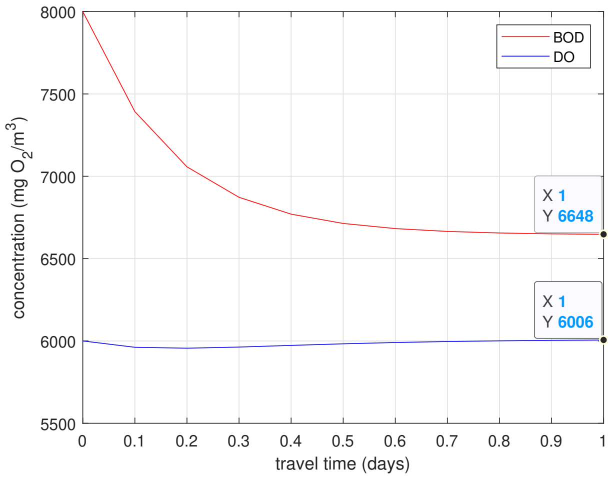

Figure 14: Case 5 (b): normal flow, low volume and low organic concentration in 24 hours of travel time

In the fifth case, it was analyzed how self-purification in the reservoir with low water volume and average flow rate occurs if there is a high input of organic matter and if there is a reduced input.

In Figure 13 it is observed that, with the entry of 20000 mg/m^3 of BOD under low volume of water, in 12 hours of travel this concentration is reduced to approximately $\frac{3}{4}$ than initially. In the case of DO, it is reduced to $\frac{4}{5}$ from the initial concentration in the same period. On the other hand, with low input of organic matter, as shown in Figure 14, the BOD has a 13 % reduction in the 12-hour period, and the DO has only 4 % of its concentration reduced in the same period.

The percentage of reduction in organic concentration, in both cases, was the same: 17 % of BOD. The difference, however, was in the dissolved oxygen consumed in the aquatic environment to perform the purification: in the first case, with high organic concentration, 22 % of the oxygen in the medium was consumed to depurate the organic matter; in the second case, less than 1 % of oxygen was consumed.

The decomposition of organic material happens more slowly in the last 12 hours of the day. Figure 13 shows that BOD ranges from 20000 mg/m^3 to 16500 mg/m^3 in one day, and the oxygen goes from 5000 mg/m^3 to approximately 3900 mg/m^3. In the scenario with low organic concentration (Figure 14), BOD drops from 8000 mg/m^3 to 6600 mg/m^3, while the DO goes from 6000 mg/m^3 to 6006 mg/m^3, or technically remains at 6000 mg/m^3. That is, after 24 hours BOD still decays at the same rate in both cases, but the dissolved oxygen is presented in a scenario more favorable to a reservoir facing a low volume if it receives less organic discharge.

It was found that the efficiency of self-cleaning is greater when there is a greater volume of water in the reservoir. In the second case analyzed (Figure 10), whose BOD input was 20000 mg/m^3, the removal efficiency of organic matter was 52.6 % for 12 hours of travel and 76.3 % for 24 hours. In the fifth case (a) (Figure 13), whose BOD input was also 20000 mg/m^3 however the volume of the reservoir was quite reduced, the organic matter efficiency of removal was 16.55 % in the first 12 hours of travel and 17.4 % for 24 hours.

Discussion

The model developed (Equations 1 and 3) has shown to be applicable to different reservoirs, since it was validated having as parameters a reservoir present in a large river and which contains several other reservoirs. Also, it was applied in a reservoir dammed in a medium-sized river and with another water residence time.

The one-dimensional model CE-QUAL-R1 is, as well as the model proposed by this study, a model based on the conservation of mass and energy. There is, both models consider mass/energy input - output, ± reactions. CE-QUAL-R1, however, divides a reservoir in horizontal layers, and so it presents a set of differential equations that makes the computational implementation less simple (Wlosinski 1985). On the other hand, the model presented at this study is quite simple to be implemented.

The model CE-QUAL-RI, although more complete than CE-QUAL-R1, presents similar restrictions to CE-QUAL-R1, that is the difficult computational implementation and a high number of parameters, which requires a greater number of laboratory analysis.

Mike models are not analyzed in this discussion because the access to their model equations and parameters require the purchase of a license, which makes their application deprecated when compared to other existing models.

Vázquez and Mokrova (2019) suggest the use of the models through GIS-MM integration. The study presents the advantages and disadvantages of using two models that could fulfill this integration function: SWAT model (Arnold et al., 2012), a model widely used for the analysis of hydrological processes. That is also based on the equation of the water balance and has, as initial parameters, the inflow and outflow of water by flow, water input by rain and water out by evaporation; and the IBER Bi-dimensional Model, used for river hydrodynamics, flood areas studies, evaluation of sediment transport, among other tools for hydrodynamic studies, also based on the equations of conservation of mass and momentum, in this case in two horizontal directions, since the model is two-dimensional.

The model is quite complete in terms of physical phenomena in the reservoir, including parameters such as friction caused by wind. However, it is a model whose application requires more advanced computational implementation, since it is a two-dimensional model, and does not consider physical-chemical and biological phenomena whose studies allow the evaluation of water self-purification, such as mineralization of organic compounds.

The model proposed by Vázquez and Mokrova (2019) proposes an easy daily application of the tool by operators and managers (once implemented), as some of the data that feeds the model could be obtained through GIS. However, the study lacks validation and, as it has been presented so far, it would not be useful to specifically study the self-purification capacity in reservoirs.

Jiang et al. (2016) use the combination of the Hydrodynamic Model of MIKE 21 coupled with the transport module (TR). They propose the use of a 2-D model to simulate the flow behavior and the contaminant convection transport in the tidal river, using the two-dimensional advection-dispersion equation. Although it is a very complete model, it is two-dimensional, which makes its computational implementation more elaborated. Besides, it presents quite many parameters to fill, which diverges from the proposal of this study, that is to present a model with few parameters and easy to apply.

The model proposed by Reartes et al. (2016) differs from the model presented in this study in terms of practicality of application due to the number of parameters it considers. The model exposed in Reartes et al. (2016) was developed by dividing the reservoir into two layers: upper and lower. The first aspect to highlight is that the Reartes et al. (2016) model has an output for water treatment in the lower layer, so it is interpreted as a model for a reservoir that has an outlet for public supply; the second point is that it has many parameters for estimating the problem, many of them specific and that require a detailed empirical study before applying the model. The model presented in this study through the Equations 1 and 3, on the other hand, requires only empirical data of flow, volume, BOD and DO.

Regarding the model applied by Tercini and Méllo Junior (2016)study used at this work to validate the model, it is an adaptation of Street & Phelps model, a model initially developed to study self-decay capability in lotic watercourses. Though, in the case of Tercini and Méllo Junior (2016), it was used residence time as a parameter to analyze self-decay rates in dammed environments. Although it is also a simple model to apply in lentic aquatic environments, it does not consider the organic matter sedimentation effect in the water quality of the reservoir's epilimnium.

Thus, when compared to other models, the model presented in this study has advantages due to its easy application, and can be used with simplicity by sectors that carry out water management, accepting the hypothesis initially posed in this study. It has, however, limitations that could be evaluated in future works. Temperature was not included as a parameter to analyze water clearance and this can lead to a less accurate analysis in months of thermal stratification. Furthermore, the *in loco* contributions of organic matter, which could provide a more detailed analysis of organic load by study point, as discussed in Figure 8, Section Results, might also be considered in future researches.

Furthermore, it is suggested the insertion of parameters that represent the decomposition in the hypolimnion and the interactions between water and sediment that result, for example, in the resuspension of organic material in the deep zone of the reservoirs. A more punctual study of the hypolimnion is extremely important to, in addition to understanding organic purification in deep areas, understand the real situation of reservoirs that receive large loads of organic material from untreated sewage; this is because the study of the epilimnion can result in the false idea that a reservoir that receives certain organic loads happens to have these organic loads mostly mineralized. Though, part of this organic load ends up being sedimented and, in the hypolimnion, faces a slower mineralization time, as the decomposition there is mostly carried out by anaerobic organisms (Wetzel, 2001).

Conclusions

Self-purification has an essential function improving the water sources quality, as observed in the simulations. In all cases analyzed, the reservoir would begin to recover its balance within the first two days of the journey, and with higher efficiency when the volume of water in the reservoir is greater.

The mathematical model developed is a suitable tool to study and evaluate the self-purification process and run-time in any reservoir. It was possible to make analysis and predictions of the organic matter and dissolved oxygen concentrations at the reservoir studied using the developed model and considering possible scenario changes.

It is shown as an innovative and relevant model to be used both in private enterprises and in the public management of watercourses to evaluate self-purification capability of any reservoir. That confirms the hypothesis initially assumed to develop this work. The effective monitoring and control carried out by the competent organizations can result in actions to improve the water bodies quality, such as incentives to increase the rates of domestic and industrial sewage treatment that are discharged in reservoirs and restoration of riparian forests.

It is pointed out the possibility of adding, for future works - since the model is passive of upgrading - the study of hypolimnium in the mathematical model, in addition to the alternative of complementing the model considering other physical-chemical and biological phenomena.

Also, for future works, it is recommended the validation and application of the developed model using interconnected reservoirs, adding model output parameters without major complications in the computational implementation.

Acknowledgments

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Relevance of Null Hypothesis Significance Testing (NHST) in biomedical sciences: sociological approach

Relevancia de las pruebas de significación de hipótesis nulas (NHST) en ciencias biológicas: enfoque sociológico

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Abstract—Significance tests play a very important role in the scientific community, and the biomedical research community is not an exception. This is due, on the one hand, to the widespread use of the test in scientific methodology and the corresponding frequency of its application in research, and, on the other hand, to the general misinterpretation of the results obtained using this method. Misunderstanding of significance testing in academia and erroneous conclusions in research, regardless of the scientific field, are at the root of the distrust of this statistical method. This article aims to give insight into the relevance of this kind of method in the biomedical field and find a theoretical explanation for this phenomenon, and subsequently regulate the correct interpretation of the null hypothesis significance test (NHST), as well as consider alternative statistical methods. In addition, some relevant empirical studies from a geographical and multidisciplinary perspective will be presented to determine the real extent of misspecification at the academic level. In this way, both practical and theoretical arguments will be applied to address the problem of NHST at multiple levels.

Keywords—Bioinformatics, Biometric, p-value, Significance tests, Statistical testing, Misinterpretation

Resumen— Las pruebas de significación desempeñan un papel muy importante en la comunidad científica, y la comunidad de investigación biomédica no es una excepción. Esto se debe, por un lado, al uso generalizado de la prueba en la metodología científica y a la correspondiente frecuencia de su aplicación en la investigación y, por otro, a la mala interpretación generalizada de los resultados obtenidos con este método. La incomprensión de las pruebas de significación en el mundo académico y las conclusiones erróneas en la investigación, independientemente del ámbito científico, están en el origen de la desconfianza hacia este método estadístico. Este artículo pretende dar a conocer la relevancia de este tipo de método en el ámbito biomédico y encontrar una explicación teórica a este fenómeno, para posteriormente regular la correcta interpretación de la prueba de significación de hipótesis nula (NHST), así como considerar métodos estadísticos alternativos. Además, se presentarán algunos estudios empíricos relevantes desde una perspectiva geográfica y multidisciplinar para determinar el alcance real de la mala especificación a nivel académico. De este modo, se aplicarán argumentos tanto prácticos como teóricos para abordar el problema de la NHST a múltiples niveles.

Palabras clave— Bioinformática, Biometría, p valor, Pruebas de significación, Pruebas estadísticas, Interpretación errónea

Introduction

Debates, warnings, prohibitions, and precautions against null hypothesis significance testing (NHST) have become commonplace in the scientific community. Moreover, criticisms of this type of statistical testing can be found regardless of the scientific field. The misinterpretation and misuse of NHST results are widely practised in medicine, biology, social sciences, and many other areas. Therefore, there are many recommendations to use alternative methods of statistical analysis, or even guidance on how to NOT interpret the results of significance tests (Wasserstein and Lazar, 2016). Moreover, there are even guidelines if one still prefers to use the p-value. All these measurements are still not able to solve the “p-value issue”. The importance of this test for biological and clinical research is not difficult to determine, as it is the gold standard for the most commonly used clinical trials (Kelter, 2020). On the one hand, the prevalence of use, and on the other hand, the prevalence of misinterpretation, prompts a more in-depth analysis of this problem. So, why do we still use this test? Firstly, most statistical software uses this test as an apriori method and secondly, just because we still learn it at universities. Thus, students are still studying NHST at the university, despite the prohibitions and restrictions of scientific and statistical societies. If for the first argument, we could still use modern software (e.g., a programming language where it is possible to perform any required analysis) and develop and promote the dissemination of machine learning algorithms in methodology with its two-stage nature: cross-validation and algorithm (Bzdok and Meyer-Lindenberg, 2018), then what about the second argument and why is it so important? In the following, we will find out the theoretical arguments for the emergence of misinterpretation. An attempt will also be made to identify whether it is related to geographical location and field of activity. We will also propose a research design that could test such theoretical arguments. The final part will discuss some alternatives to the p-value test, their limitations and if machine learning can help by neutralizing misinterpretation in research.

NHST IN BIOMEDICAL RESEARCH

The problem of not understanding the concept of p-value is dramatic because it affects directly reproducibility of scientific research and raises growing concerns about the credibility of claims of new findings based on ’statistically significant’ results (Benjamin et al., 2017, p. 5). Therefore, Szucs and Ioannidis (2017) investigated the replication success and reported poor replication rates in psychology, and added that we may expect even lower replication rates in cognitive neuroscience. Such ’findings’ can be found in studies that determine the effect of a drug being tested by comparing this effect between control and treatment groups. They are also used in finding the association of spillover patterns with the disease of interest. These are just some examples of the types of studies in which the NHST is generally accepted and possibly used right now.

According to Gao (2020), reproducibility is not the only harmful consequence. He added, that the p-value problem can also impact treatment choices in medical practice and model specification in empirical analysis (Gao, 2020, p. 1). Moreover, Ioannidis (2019) in his paper the titled "What Have We (Not) Learnt from Millions of Scientific Papers with P Values?"provides more details about publication issues and why many studies (including biomedical literature) are debatable.

So, we just defined why exactly the p-value problem is relevant for the biomedical field. The next relevant point is how actually biomedical scientists are aware of this problem and how competent they are in knowing the main concepts of NHST. Unfortunately, empirical research does not yield positive results on this issue. Therefore, Windish et al. (2007) demonstrated in their multiprogram survey that medicine residents gave a total overall of 41.4 per cent correct answers in a proposed program on statistical interpretation, and the rate for correct interpretation of p-value is 58.8 per cent (53.0-64.6) (Windish et al., 2007, p. 1014). These 58.8 per cent answered correctly (according to the design of the survey) on the question about the interpretation of p >0.05. There were four possible answers (Windish et al., 2007):

a. The chances are greater than 1 in 20 that a difference would be found again if the study were repeated.
b. The probability is less than 1 in 20 that a difference this large could occur by chance alone.
c. The probability is greater than 1 in 20 that a difference this large could occur by chance alone.
d. The chance is 95 per cent that the study is correct.

At this point, it is necessary to add the following quote:

"58.8 per cent of the residents selected choice c which was designated by the authors as the correct answer. The irony is that choice c is not correct either. In fact, none of the four choices is correct. So, not only were the residents who picked choice c wrong but also the authors as well. Keep in mind, the paper was peer-reviewed and published by one of the most prestigious medical journals in the world."(Gao, 2020, p. 12)

This study was carried out 15 years ago. It is logical to assume that the situation in the biomedical field has changed due to different recommendations, guidelines and bans in some journals. A recent Swedish study very clearly refutes this. (Lytsy et al., 2022) conducted a study on understanding the concept of significant testing among the PhD students with medical and statistical and/or epidemiological backgrounds. Results: correct answers to addressing both questions, that no statistically significant result can be derived either as proof or as a measure of hypothesis probability, were given by 10.7 per cent of doctoral students and 12.5 per cent of statisticians/epidemiologists (Lytsy et al., 2022).

Thus, the problem is clearly a global one. For a more detailed analysis, it is necessary to establish how global the problem is, i.e. whether it goes beyond the borders of Sweden and beyond the borders of statisticians and medics.

GEOGRAPHICAL AND MULTIDISCIPLINARY DIMENSIONS

Unfortunately, Sweden is not alone in the widespread misunderstanding of the NHST concept at the academic level. The following empirical results support this argument. Therefore, they demonstrate that the problem of understanding and interpreting the significant test is not local and, furthermore, has little to do with the academic specialization of the respondents. Surveys in Germany, China, Spain, Italy and Chile have shown that not only students but also academic teachers with sufficient experience in university teaching commit errors in carrying out a significant test. In addition, misinterpretation extends beyond the biological and medical sciences to other fields. The following will briefly discuss the results of these studies.

German survey

Suppose you have a treatment that you suspect may alter performance on a certain task. You compare the means of your control and experimental groups (say 20 subjects in each sample). Further, suppose you use a simple independent means t-test and your result is (t = 2.7, d.f. = 18, p = 0.01). Please mark each of the statements below as "true" or "false". "False" means that the statement does not follow logically from the above premises. Also note that several or none of the statements may be correct.

1) You have absolutely disproved the null hypothesis (that is, there is no difference between the population means). [ ] true / false [ ]

2) You have found the probability of the null hypothesis being true. [ ] true / false [ ]

3) You have absolutely proved your experimental hypothesis (that there is a difference between the population means). [ ] true / false [ ]

4) You can deduce the probability of the experimental hypothesis being true. [ ] true / false [ ]

5) You know, if you decide to reject the null hypothesis, the probability that you are making the wrong decision. [ ] true / false [ ]

6) You have a reliable experimental finding in the sense that if, hypothetically, the experiment were repeated a great number of times, you would obtain a significant result on 99% of occasions. [ ] true / false [ ]

Figure 1: The Questionnaire of the survey of Haller and Kraus (Haller and Kraus, 2002, p.5)

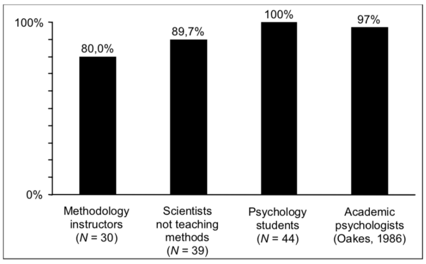

Figure 2: Percentages of participants in each group who made at least one mistake, in comparison to Oakes' original study (1986) (Haller and Kraus, 2002, p.7)

The main purpose of Haller and Kraus' study was to determine whether psychology students and teachers can interpret significance tests correctly and give corresponding pedagogical guidelines for the correct use of NHST Haller and Kraus (2002). They offered in their paper a discussion about possible reasons for misinterpretation. As result, they determined that statistical textbooks and statistical instructors could be the cause. The researchers presented a questionnaire, in which they collected questions related to the NHST and this served as a sufficient measure to check the level of knowledge in the application of the NHST by students and teachers of psychology. In addition, they divided the participants into three groups: teachers of methodology, like professors with NHST accreditation, research psychologists (who do not teach), and psychology students. The questionnaire is presented in Figure 1. As the correct answer to all the questions is 'False', they also added an explanation of why this is false for each of the questions. Furthermore, they compare the results of their survey with those of the Oakes survey, which is also very important for the discussion on the relevance of the NHST. The Haller and Kraus study shows that 16 years after the Oakes study, which shows the highest rate of misinterpretation of significance tests by academic psychologists, the problem is still relevant (see Figure 2).

Chinese survey

The second perceptive study is also one of the most recently published studies in this field (Lyu et al., 2020). Compared to the study by Haller and Kraus (2002), this study has several improvements. They examined how students interpret the significance test and the confidence interval, the main alternative to the significance test. Their study showed that most respondents, even those with academic degrees, regardless of the field of study and career stage, were not able to interpret P values and confidence intervals accurately. Another empirical advantage is the measure of respondents' confidence in their judgement, and how confident they are in their answers. Unfortunately, there is no good result here either. Most respondents are confident in their answers, which makes the problem of misinterpreting the P-value even more dangerous.
1479 respondents took part in this survey. The questions for the NHST are similar to the questionnaire by Haller and Kraus. The questionnaire for the CI is relatively new and was taken from their previous studies. The structure and questions of this questionnaire are very plausible and can provide a good measure for interpreting the CI. They also added a second version of the questionnaire: one scenario with a significant outcome, p >0.05, and a second scenario without a significant outcome, p <0.05. One additional question measuring respondent confidence was also added for each question in both parts of the questionnaire. However, they did not explore why respondents were confident in their decisions. The results showed that 89 per cent of respondents made at least one error when interpreting the P-value and 93 per cent of respondents made at least one error when interpreting the CI. This may mean that they do not pay enough attention to CI, but rather to NHST, which most of them also interpret incorrectly. This study is also crucial for assessing the 'magnitude of the tragedy', as it not only presents results from psychologists (and social scientists) but also from respondents from other fields (see Figure 3 and Figure 4). At this point, we can conclude that the misinterpretation of NHST and CIs is a multidisciplinary problem. Moreover, even mathematicians and statisticians have not shown acceptable results for either the non-significant scenario or the insignificant scenario. This fact should motivate the whole scientific community to continue the 'p-war', especially at the academic level.

Spanish survey

The third survey Badenes-Ribera et al. (2015) presents the results of a survey of Spanish academic psychologists and methodology teachers (n = 418). Interestingly, the respondents' average length of service as university professors is 14.16 years. Thus, their conclusion is based on the competence of psychology teachers rather than on the knowledge level of the students. This study tested whether Spanish academic psychologists have a proper understanding of the concept of p-value. The researchers divided their questionnaire into four parts. Each part includes questions concerning a particular type of delusion: inverse probability, replication, effect size and clinical significance fallacies. Moreover, they concluded that the first type, reverse probability fallacy, was the most common among respondents. The fact that academic psychologists have such a high percentage of errors in the questionnaire showed that the high percentage of misinterpretations may lie not only

	Statement (significant scenario)	Science $N = 133$ (9%)	Eng/Agr. $N = 72$ (5%)	Medicine $N = 69$ (5%)	Economics $N = 93$ (6%)	Management $N = 51$ (3%)	Psychology $N = 125$ (8%)	Social Science $N = 111$ (8%)	Math/Statistics $N = 105$ (7%)	Average $N = 759$ (51%)
p value (significant)	(a) You have absolutely disproved the null hypothesis.	53%	53%	49%	60%	63%	50%	59%	44%	53%
	(b) You have found the probability of the null hypothesis being true.	58%	62%	52%	44%	55%	59%	45%	32%	51%
	(c) You know, if you decide to reject the null hypothesis, the probability that you are making the wrong decision.	53%	62%	51%	67%	71%	77%	67%	70%	65%
	(d) You have a reliable experimental finding in the sense that if, hypothetically, the experiment was repeated a great number of times, you would obtain a significant result on 99% of occasions.	62%	54%	64%	63%	53%	42%	59%	48%	55%
	Total (endorsed at least one statement)	93%	90%	90%	92%	94%	95%	95%	88%	92%
CI (significant)	(a) There is a 95% probability that the true mean lies between .1 and .4.	56%	53%	52%	60%	63%	66%	67%	33%	56%
	(b) If we were to repeat the experiment over and over, then 95% of the time the true mean falls between .1 to .4.	59%	56%	54%	54%	51%	54%	59%	48%	55%
	(c) If the null hypothesis is that there is no difference between the mean of experimental group and control group, the experiment has disproved the null hypothesis.	57%	53%	49%	53%	59%	31%	48%	40%	48%
	(d) The null hypothesis is that there is no difference between the mean of experimental group and control group. If you decide to reject the null hypothesis, the probability that you are making the wrong decision is 5%.	62%	53%	48%	66%	63%	70%	56%	58%	60%
	Total (endorsed at least one statement)	97%	93%	93%	96%	98%	94%	94%	88%	94%

Figure 3: Percentage of misinterpretation of p values and CIs for each statement (significant scenario) (Lyu et al., 2020, p.5)

	Statement (nonsignificant scenario)	Science $N = 114$ (8%)	Eng/Agr. $N = 79$ (5%)	Medicine $N = 61$ (4%)	Economics $N = 71$ (5%)	Management $N = 44$ (3%)	Psychology $N = 147$ (10%)	Social Science $N = 106$ (7%)	Math/Statistics $N = 98$ (7%)	Average $N = 720$ (49%)
p value (non-significant)	(a) You have absolutely proved the null hypothesis.	63%	57%	48%	48%	55%	54%	53%	43%	53%
	(b) You have found the probability of the alternative hypothesis being true.	57%	43%	54%	42%	48%	40%	49%	34%	45%
	(c) You know, if you decide not to reject the null hypothesis, the probability that you are making the wrong decision.	54%	56%	64%	65%	70%	63%	59%	55%	60%
	(d) You have an unreliable experimental finding in the sense that if, hypothetically, the experiment was repeated a great number of times, you would obtain a significant result on 21% of occasions.	61%	48%	43%	42%	43%	29%	45%	32%	42%
	Total (endorsed at least one statement)	87%	9%	82%	90%	93%	84%	87%	78%	86%
CI (non-significant)	(a) There is a 95% probability that the true mean lies between -.1 and .4.	62%	54%	62%	61%	55%	69%	63%	33%	58%
	(b) If we were to repeat the experiment over and over, then 95% of the time the true mean falls between -.1 to .4.	53%	49%	52%	56%	61%	48%	60%	53%	53%
	(c) If the null hypothesis is that there is no difference between the mean of experimental group and control group, the experiment has proved the null hypothesis.	54%	44%	61%	46%	43%	46%	50%	37%	48%
	(d) The null hypothesis is that there is no difference between the mean of experimental group and control group. If you decide not to reject the null hypothesis, the probability that you are making the wrong decision is 5%.	52%	58%	51%	51%	68%	53%	63%	45%	54%
	Total (endorsed at least one statement)	95%	92%	92%	89%	98%	89%	93%	85%	91%

Figure 4: Percentage of misinterpretation of p values and CIs for each statement (non-significant scenario) (Lyu et al., 2020, p.6)

in the educational program but also arise from the lack of competence of academics in the application of statistical methods.

Chilean-Italian study survey

Another study Badenes-Ribera et al. (2016), which is a replication of the previous one, has not shown any satisfactory results as well. Only two parameters were changed: the geography of the survey (Italy and Chile instead of Spain) and the response scale (correct answer scale instead of true/false). The questionnaire also includes questions on inverse probability fallacy, replication fallacy, effect size fallacy, and clinical significance fallacy. Respondents in this survey reported more correct answers compared to the previous study (Badenes-Ribera et al., 2015), what they attempted to explain by differences between countries. However, they did not provide any strong arguments to prove this and in any case, the level of misinterpretation is still high, especially for academics. This way, both studies force us to personalise the educational strategies and competencies of academics (in the field of statistical methods) in their conclusions.

The previously mentioned studies are essential for the discussion about the failure of significant tests in biomedical sciences. Moreover, the results of the second study showed that this problem should be addressed by medical scientists as well as other scientists (natural science, mathematics/statistics, management, sociologists, etc.). The third and fourth studies concluded that this problem has little or no correlation with the location of the survey. All of the countries described above did not have satisfactory survey results: USA, Germany, China, Spain, Chile, and Italy. Thus, the misinterpretation of the p-value is not local but global. However, none of these studies provides a theoretical explanation for this phenomenon. That is why a new empirical study is warranted. These studies (and others in the field) explain the roots of the misunderstanding of NHST from a statistical or methodological perspective, but not from a cognitive perspective. Analyzing the studies cited above, it becomes clear at what stage of interpretation an error occurs methodologically and even how to categorize such misinterpretation, but there is still no understanding of why this happens so systematically. Understandably, the type of misinterpretation is important for further analysis and work on preventive measures, but it is still not enough to reduce the number of misinterpretations and misuses of the p-value. In the next section, we will propose some theoretical arguments that may explain this phenomenon and encourage new research in this area by applying a cognitive perspective to the problem.

Source theory as theoretical argumentation

The preceding empirical outcomes motivate the idea that there is something behind methodological errors. Even if we have found the roots of the errors, learned how to classify them, developed guidelines for avoiding these errors, and achieved bans at a high scientific level, because of the 'p-war' in the scientific community, NHST is still used in research and in university courses. Moreover, even if we 'win' this war, will the problem be solved, or will other misunderstandings of statistical methods arise? To try to explore this, we need to go back to the starting point by applying a sociological approach. It is logical to assume that if we are going to use statistical methods, we first need to learn them. The process of learning these methods in biology, medicine, and other scientific fields begins at universities if one considers science. Consequently, empirical evidence shows that often professors cannot provide evidence of 100 per cent understanding of concepts related to p-value resulting in some of them teaching their students about NHST without conveying a fully correct interpretation of these concepts. Potentially, we can find the results of research conducted by scientists without a complete understanding of NHST interpretation in medical trials, therapeutic surveys, and other high-impactful studies. Now, is there anything special about this transit between professor and student?

Such an explanation should contain a theoretical mechanism that describes the influence of the teacher (the object that provides the primary information about the ST) on the student (the object that perceives this information). In this relationship, we must determine the origin of the error. So, why don't students check the validity of the learning material? It is logical to assume that teachers have a high degree of trust and authority on the part of students and that the information and knowledge they provide are perceived as reliable. We assume that this problem arises when transferring knowledge from teacher to student. This means that the mistake and the root of misinterpretation arise at this very moment. Thus, students receive information about significant tests without having to verify it and use this information in their research or teaching other students in their careers. This is a potential scenario that can lead to such global misinterpretation in the application of statistical methods.

Thus, the focus of this section is on source theory, a theory that derives expectations based on characteristics and competence. Source theory belongs partly to the Expectation-States Theory, which is not a concrete theory or a paradigm, but rather a research program that units many other theories to explain the relationship between performance expectations and social influence (Kalkhoff and Thye, 2006).

The student trusts their teacher and has no doubts about their competence. The main question of this interaction process is: why do students trust their teachers in the first place? To answer this question, we use a source theory approach, where we point to the teacher as a source of evaluation for the student. Savage and Webster (1972) explained source theory as a combination of two theoretical concepts. The first concept of the 'Mirror Self' from Cooley and Mead postulates:

"Evaluations from a significant other, to use Sullivan's term, will predictably be accepted by the individual, whereas the opinions of others (with uncertain characteristics) are likely to be ignored" (Savage and Webster, 1972, p. 317)

This statement should be accompanied by an explanation of the concept of the 'significant other':

"Cooley located such significant others primarily in families and peers but there are individuals who have the right to evaluate the performance of others in many kinds of more formal settings as well; employers have the right to evaluate employees, teachers the right to evaluate students." (Berger et al., 1983, p. 24)

According to Savage and Webster (1972), the second theoretical concept is a crucial claim of expectation states theory, which argues that 'many regularly reported observable behaviours among the members of problem-solving groups, such as an unequal number of chances to perform, evaluations of performances, the likelihood of performing, and rejection of influence, may be explained if one postulates the existence of expectation states or cognitive beliefs about the ability of each member of the group' (Savage and Webster, 1972, p. 318). In sum, they make a statement of the Source theory such as: 'an individual whom a high ability evaluator evaluates will often believe him and form an expectation state based on those evaluations, while an individual evaluated by a low ability evaluator will usually ignore him; and an individual who holds high self-expectations will be more likely to reject influence than an individual who holds low self-expectations (Savage and Webster, 1972, p. 318). In addition to the claim that our phenomenon is perfectly explained by resource theory, it should be added that Berger et al. (1983) also note the importance of having rules in this interaction:

"The more rules there are, the less likely incongruence is to arise in the first place." (Berger et al., 1983, p. 36)

Thus, it is more likely to notice incongruence between the expectation state and authority in informal interaction than informal. For example, the interaction between teacher and student is full of formality, e.g., university regulations, official appeals, subordination, etc.

Since we have discussed all the essential components of the Source theory, we can then describe the teacher-student interaction in terms of this theory. In a possible situation, the student has high expectations of their teacher, based on the teacher's status, and so the teacher acts as a 'significant other' to the student. Therefore, as a significant other, the teacher can legitimately judge the student's performance and the student accepts this. Having rules in this interaction makes it less likely that the teacher will lose their authority from the student's perspective. In addition, the teacher influences the student and the student accepts this influence and applies it to the learning process. Thus, if a teacher transmits incorrect knowledge about NHST to a student, the student is more likely to believe them because of their high expectations and less likely to test the validity of the learning material. However, this possible theoretical scenario excludes cases in which a teacher provides the correct concept of NHST to their students. It also excludes cases where students are less motivated to learn and make no significant effort to understand the learning material. In this theoretical scenario, we focus only on cases where students are motivated to understand the concept of NHST and teachers provide incorrect knowledge.

Moreover, this explanation is not intended to discredit professors. On the contrary, this theoretical argumentation makes it clearer how powerful real pedagogical influence is and what critical consequences it can lead to. In addition, it can help to understand the fundamental role of critical thinking skills in an academic environment.

PROPOSAL OF RESEARCH DESIGN AND SURVEY

The theories described above have a sufficient number of experimental tests (Thye and Kalkhoff, 2009) and therefore we will not operationalize the theory in our study and will use it as a given. To derive the hypothesis, we need to formulate two premises. First, in line with empirical research, we recognise that teachers can teach students an inaccurate conception of NHST. [assumption 1]. Secondly, given this, we also believe that if teachers provide misinformation, some students may verify it from other sources due to low trust, in which case we expect better results from these students. Thus, students' critical perception is the second precondition for the hypothesis [assumption 2]. On this basis, we expect worse test results from students who trust their teachers more (according to the theory). Based on the theoretical discussion and the two premises, the following hypothesis will be derived:

Given that the level of competence of an instructor in NHST is low, the more a student will trust this instructor in statistics, the more this student will commit errors in NHST.

To determine the association between the effect of teacher status and the level of conceptual understanding of NHST, we derive two variables: a dependent variable and an independent variable. The dependent variable refers to the number of correct answers given by the student, and the independent variable is the student's confidence in the competence of their teacher in statistical knowledge. To investigate the level of underestimation of NHST, we propose six questions contained in our questionnaire (see appendix). The sum of the correct answers (min = 0, max = 6) is the measure of the dependent variable. To measure the independent variable, trust, we first need to define the meaning of trust in the context of our study. 'Trust' means that the student has full confidence in the competence of their teacher and does not check the verification of the learning material. It will also ask whether the teacher (if they are the source of knowledge about the NHST for the student) influences the student's level of knowledge. After each test question, this question will be asked to track which questions students make the most mistakes in. Also, before the test questions, we will ask them how they assess the competence of their statistics instructor who teached them the basics of NHST. This 'trust' will serve as the independent variable. Logically, we will accept the hypothesis if we find a sufficient number of students who score well on the significant test questionnaire and most of their questions were not influenced by their teacher (Figure 5, quadrant II). We will also accept it if we get a sufficient number of students who have poor scores on the NHST test, but most of their questions were influenced by their teacher (Figure 5, quadrant IV). The case when we reject the hypothesis is when the survey shows a large number of students who did not commit errors in the NHST test and most of their questions were influenced by the teacher, and the reverse (Figure 5, quadrants I and III).

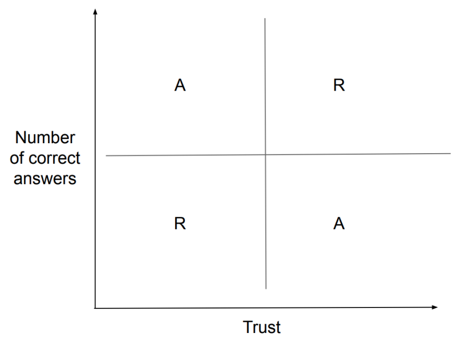

Figure 5: Accept or reject the hypothesis

To test the hypothesis and get a generalizable knowledge about the causes of a misconception of NHST in the educational system, we suggest the following study design. Since the previous studies provide a questionnaire to measure the level of knowledge in NHST, we will continue to apply a comparable design (see appendix). The questionnaire for NHST is taken from the study Haller and Kraus (2002). The confidence intervals questionnaire was compiled by us (see appendix, sections 4-5). Before each of both test-questionnaire starts, we will ask students how they rate the competence of their lecturer in statistics who teaches them the basics of significance testing/confidence intervals, using the Likert scale. Very competent means that they fully believe in the plausibility of the material taught, incompetent means, that they do not believe in the plausibility of the material taught and check it with other sources. In addition, for each question of the test questionnaire they have to answer whether their answer is influenced by the knowledge they have acquired with the help of the teacher. We also accept the advice from Badenes-Ribera et al. (2016) to use the three-response-format «True/False/Don't know»:

"By not asking to explicitly classify statements as either true or false, it is not possible to differentiate omissions from items identified as false. A three-response format (True/False/Don't know) would have been far more informative since this would have also allowed identifying omissions as such." (Badenes-Ribera et al., 2016, p. 8)

and we have added it to our questionnaire in form of 'True/False/Neither/nor'. Our survey is designed for students, professors, and tutors to check the level of knowledge of each of these categories. However, in order to test the hypothesis, its acceptance or rejection, we will take the results only of those who indicated their student status in the first part of the questionnaire and chose lecture as a source of information for ST. Thus, for a valid analysis, the group of students must be matched to their statistics teacher. This means that we should first measure the level of NHST knowledge of the students (who choose lecture as a source of NHST knowledge), their assessment of the teacher's competence, and then compare it with the level of NHST knowledge of this teacher.

The results of this analysis can potentially be compared with the results of previous studies to produce a complete conclusion.

Discussion: Practical alternatives to NHST

According to Ioannidis (2019), there is a great call for resolute action not only on the part of statisticians but also on the part of the entire scientific community. The theoretical arguments presented above are not intended to change the relationship between teachers and students, but rather to demonstrate how critically students perceive information from their professors and to draw attention to the widespread misinterpretation of NHST from a cognitive and pedagogical perspective. Now, methodologically speaking, the main mistake many researchers make is to use the NHST as a method to draw definitive conclusions. In fact, this test serves as a filter tool. Moreover, a non-significant result does not mean that the study is meaningless (MacGillivray, 2019). If a drug study shows a p >0.05, it may still have medical relevance. In other words, this study may contribute to the development of many studies in other areas of medicine. Thus, the P-value is not a measure of success or failure:

"P values are neither objective nor credible measures of evidence in statistical significance testing. Moreover, the authenticity of many published studies with p <0.05 findings must be called into question." (Hubbard and Lindsay, 2008, p. 81)

If the results are not properly analyzed, the research provides incorrect or incomplete information, and this starts a chain of incorrect data that grows like a snowball. This is why proper interpretation is vital for every field of research. In our discussion, we rely on the following statement about the correct interpretation of the NHST for further analysis:

"The p-value is not the probability of the null hypothesis; rejecting the null hypothesis does not prove that the alternative hypothesis is true; not rejecting the null hypothesis does not prove that the alternative hypothesis is false; and the p-value does not give any indication of the effect size. Furthermore, the p-value does not indicate the replicability of results. Therefore, NHST only tells us about the probability of obtaining data which are equally or more discrepant than those obtained in the event that H0 is true (. . .)" (Badenes-Ribera et al., 2016, p. 7)

Nevertheless, it is also possible to choose the right strategy for teaching statistical methods. For example, Haller and Kraus (2002) proposed in their paper four steps to avoid misunderstanding by students. The main idea is to present NHST with terms for conditional probabilities, such as Bayes' rule (Haller and Kraus, 2002, p. 10):

"Teach students the underlying idea of the Bayesian inference approach: Considering p(H | D) To find out the probability of a hypothesis (H) given data (D), we can apply Bayes' rule: Considering p(H | D):

$$p(H|D) = \left((p(D|H)p(H))/(p(D|H)p(H) + p(D|H)p(H)\right)"$$

Moreover, (Kelter, 2020) proposed in his paper to use of special software (JASP) as an alternative to NHST, which is based on Bayesian inference and makes it more clear for scientists to rehearse.

On the one hand, this strategy could definitely increase understanding of the concept of NHST and positively influence the quality of the application. On the other hand, a deductive approach is needed not only for NHST but also for teaching other statistical methods.

Thus, a second important risk factor is the lack of critical thinking among students (Santos, 2017). The idea that critical thinking should be an important aspect of science education is widely recognised (Tanti et al., 2020) For example, the National Science Education Standards sets as one of its goals the promotion of science as a research activity (National Academy of Sciences, 1996). This goal includes numerous items that focus on critical thinking, such as 'identifying assumptions, applying critical and logical thinking, and considering alternative explanations; analysing events and phenomena firsthand and critically examining secondary sources; testing the reliability of the knowledge they generate; and critical skills in analysing arguments by reviewing current scientific understanding, weighing evidence, and examining logic to decide which explanations and models are best' (Bailin, 2002, p. 361)

As already mentioned, there are also other alternatives to the significance test: confidence intervals, effect sizes and algorithms of machine learning. We will briefly discuss their advantages below.

Compared to the p-value, the CI provides a reasonable estimate of the size of the effect in the population, indicates the precision or reliability of the estimate by the width of the interval, and the CI focuses on whether the results are meaningful to the population rather than aiming for a statistically significant value (Hubbard and Lindsay, 2008, p. 81). Furthermore, p-values do not provide information about the size of the effect. A statistical test will almost always show a significant difference in a larger sample unless there is no effect at all, i.e. the effect size is zero (Sullivan and Feinn, 2012, pp. 279–280). Sullivan and Feinn (2012) concluded that an estimation of the effect size is required before the start of the study to calculate the number of subjects needed to avoid a type II error (Sullivan and Feinn, 2012, p. 279). Thus, large but insignificant effects may lead to further searches with a higher power, while trivially small effects that are significant due to large sample sizes may warn researchers to potentially overestimate the observed effect (Fritz et al., 2011, p. 2). For a detailed explanation of alternative techniques for different types of studies, such as vitro and animal studies, genetic studies, equivalence and noninferiority trials and also descriptive and diagnostic studies, see Schmidt et al. (2018)

However, despite numerous recommendations and advice on the use of alternatives, this does not actually seem to be useful or resultant. According to Ioannidis (2019), empirical data suggest that across the biomedical literature (1990–2015), when abstracts use P values 96 per cent of them have P values of 0.05 or less. The same percentage (96 per cent) applies to full-text articles. Among 100 articles in PubMed, 55 report P values, while only 4 present confidence intervals for all the reported effect sizes, none use Bayesian methods and none use a false-discovery rate. (Ioannidis, 2019, p. 20)

Moreover, there are great studies and literature, which recommend decent interpretations of the p-value: J.Benjamin and Berger (2019); Amrheinm et al. (2017); Colquhoun (2017); Benjamin et al. (2017); Gliner et al. (2001); Zhang and Wu (2022); Greenland et al. (2016), if one argues that alternative methods or alternative teaching strategies are quite complex.

The final argument is machine learning and its prominent role in the debate about alternatives to the significance test. Bzdok and Meyer-Lindenberg (2018) describe in their paper how machine learning procedures and approaches can eliminate the inductive problems associated with NHST. Such approaches allow multiple outcomes at once, which is very important when investigating the effects of drugs or disease-induced patterns; they also offer solutions on how to treat multi-class predictions instead of one isolated model; as already mentioned, machine learning algorithms include a two-step procedure, which means that the original data set is split into subsets for testing and learning to see the accuracy of the chosen model, this split can be applied to further research (Bzdok and Meyer-Lindenberg, 2018) or generally help build more accurate models by tuning and comparing different models simultaneously. Moreover, machine learning is particularly advanced in predicting modelling. Predictive modelling is very relevant to biomedical research, as it can help health professionals by making predictions for patients at the individual risk of developing a disease (or disorders) and further assisting them with diagnostic tasks (Steyerberg et al., 2018). In addition, they can help healthcare professionals write prescriptions, better assess patients' conditions, and improve their lifestyles.

Indeed, machine learning has many chances to correct inductive problems by understanding and applying statistical methods in scientific research. The biggest drawback here is the (seeming) complexity of algorithms and programming languages for many scientists. However, establishing machine learning algorithms could lead to an increase in the overall quality of research due to their more accurate interpretation of results

Conclusion

The problem of the scandalous p-value does not rest on the rock of faith: it is not likely or less likely. It has become a 'physical law' in the scientific community. Consequently, it also strongly influences the development and quality of research in the biological and medical fields. Furthermore, we can observe that despite the guidelines, recommendations, bans, and other measures against p-value as the only 'source of truth', the trend towards a decline in its use is not expected. In this regard, we accepted the challenge to investigate the roots of this problem not only from a statistical or methodological perspective but also to apply an approach from other sciences, in our case, a sociological approach. We have analytically derived some arguments that point to the lack of a unified pedagogical strategy for teaching NHST to students of various disciplines. We also proposed a potential research design that could test the validity of the sources logically derived from the theory. Of course, we would not see $p < 0.05$ as an indicator of the success or failure of such a study. If such arguments hold true, this may encourage the scientific community to take action to regulate the limits of p-value strength not only at the publication level, but also at the academic level. We also believe that combining scientific approaches from different disciplines (biomedical and sociological in our case) can bring much more benefit and power to regulating problems in the scientific community, be they of p-value or otherwise.

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I would like to express my gratitude to Dr. Stephan Poppe for fruitful discussions and advices.

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Proposal of the questionnaire

1.1. Select your status:

- Student
- Academic teacher
- Tutor
- None above

1.2. Select your discipline:

- Medicine
- Biology
- Psychology
- Social science
- Other ()

1.3. Select your university:

- University ()
- Other ()

1.4. Have you participated in statistical courses (lectures, seminars or similar)?

- Yes
- No

1.5. How long ago was the last course?*
*(triggered by Q1.4: yes)

- less than 1 year
- between 1 and 2 years
- more than 3 years

1.6. Have you acquired statistical knowledge yourself (outside of courses)?*
*(triggered by Q1.1: Academic teacher)

- Yes
- No

2.1. How would you rate your knowledge of significance testing?

✺ very good ✺ good ✺ partially ✺ not good ✺ no knowledge

2.2. Where did you acquire your knowledge of the basics of significance testing?

- Classes (lectures)
- Internet sources (internet courses)
- Other (please specify here)

2.3. How would you rate your knowledge of significance testing?*

(Very competent means that you fully believe in the plausibility of the contents taught,
no competence means that you do not believe in the plausibility of the contents taught and that you check them on your own)
*triggered by Q2.2: Classes(lectures))

✺ very competent ✺ competent ✺ poor competent ✺ no competence at all

2.4. Do you know about any alternatives to the significance test for testing hypotheses?

- Yes
- No

2.5. What would you use to show an effect in your data? (Multiple answers possible)

- Null hypothesis significance testing (NHST)
- Confidence Interval
- Other alternatives (effect sizes, power test)
- Other()

Suppose you apply a simple t-test for independent samples to examine a mean difference between an experimental and a control group. The difference between the groups is significant at the 1% level (more precisely: $t = 2.7$, $df = 18$ degrees of freedom, $p = 0.01$). Please mark each of the following statements as "true", "false" or "neither/nor". "Neither/nor" means that the statement does not follow strictly logically from the above premises.

3.1. It is clearly proven that the null hypothesis (that there is no difference between the population targets) is false.

3.2. The probability of the null hypothesis being true has been found.

3.3. It has been proven that your alternative hypothesis (that there is a difference between the population targets) is true.

3.4. The probability that the alternative hypothesis is correct can now be derived.

3.5. If one now decides to reject the null hypothesis, then one now knows the probability that this decision could be wrong.

3.6. The experimental finding is reliable in the sense that you would get a significant result in 99% of the cases if you repeated the experiment very often.

- True
- False
- Neither/nor

(asked after each question):
Does the knowledge you have acquired with the help of your teacher(s) influence your answer?

- Yes
- No
- I had no teacher

4.1. How would you rate your knowledge of confidence interval?

✺ very good ✺ good ✺ partially ✺ not good ✺ no knowledge

4.2. Where did you acquire your knowledge of the basics of confidence interval?

- Classes (lectures)
- Internet sources (internet courses)
- Other (please specify here)

4.3. How would you rate your knowledge of confidence interval?*

(Very competent means that you fully believe in the plausibility of the contents taught,
no competence means that you do not believe in the plausibility of the contents taught and that you check them on your own.)
*(triggered by Q4.2: Classes(lectures))

✺ very competent ✺ competent ✺ poor competent ✺ no competence at all

Suppose you are working at a research institute and you are dealing with the issue of gender pay gap. You sample 100 people to estimate the pay gap.
They observe an average difference in hourly earnings of 4.5€, with a symmetric 95% confidence interval of [3.8, 9.4].

Appendix

5.1. It is clearly proven that the null hypothesis (that there is no difference between the population targets) is false.

Appendix 1 (see bellow)

- True
- False

5.2. The probability of the null hypothesis being true has been found..

- wider
- narrower

5.3. It has been proven that your alternative hypothesis (that there is a difference between the population targets) is true.

- wider
- narrower

5.4. The probability that the alternative hypothesis is correct can now be derived.

- True
- False

5.5. If one now decides to reject the null hypothesis, then one now knows the probability that this decision could be wrong.

- True
- False

(asked after each question):
Does the knowledge you have acquired with the help of your teacher(s) influence your answer?

- Yes
- No
- I had no teacher

Persistence condition on mobility parameters for obligate-migration populations

Condición de persistencia en los parámetros de movilidad para poblaciones de migración obligada

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Abstract—Animal species change their residence places due to the high temporal variability in the availability of resources. Thus, animals move to sites with higher productivity and they search environments with the necessary resources that allow it to develop the different stages of their life cycle. Here, we present a simple mathematical model that determines a no extinction condition (stable oscillation) in terms of the movement parameters for obligate-migration populations. We understand the obligate-migration as a movement seasonally predictable in distinct locations, which is associated with patterns of resource distribution.

Keywords—Biology of migration, Obligate-migration, Mathematical model, Population persistence

Resumen—Las especies animales cambian de lugar de residencia debido a la alta variabilidad temporal en la disponibilidad de recursos. Así, los animales se trasladan a sitios de mayor productividad y buscan ambientes con los recursos necesarios que les permitan desarrollar las diferentes etapas de su ciclo de vida. Aquí, presentamos un modelo matemático simple que determina una condición de no extinción (oscilación estable) en términos de los parámetros de movimiento para poblaciones de migración obligada. Entendemos la migración-obligatoria como un movimiento estacionalmente predecible en distintas localidades, que está asociado a patrones de distribución de recursos.

Palabras clave— Biología de la migración, Migración-obligada, Modelo matemático, Persistencia de la población

Introduction

A From a mechanistic point of view, animal species could be classified as residents or migrants Newton (2012); Watts et al. (2018), according to mobility of their populations to optimize their reproduction and survival Boyle (2017); Dingle and Drake (2007). Thus, resident populations are composed of individuals that perform all stages of their life cycle within a single geographical area, finding in it the necessary resources for their development and reproduction Dingle and Drake (2007); Cornelius et al. (2013b); Newton (2008). In contrast, those needing two or more geographical areas to these biological processes are called migrants Chapman et al. (2011); Newton (2012); Pedler et al. (2014); Roshier et al. (2008, 2006); Singh and Leonardsson (2014).

Due to spatial and temporal fluctuation in resource availability beyond of tolerance ranges of species, they are forced to make changes in their residence places, moving to sites of greater productivity and appropriate environment for their optimal development Boyle (2017); Cornelius et al. (2013b); Romero and Wingfield (2015); Stojanovic et al. (2015). The generated diffusion by migratory movements together to geographic and temporal variations in the resource availability, produce corresponding changes in behavioral and physiological characteristics Chapman et al. (2011); Cornelius et al. (2013a); Fryxell and Sinclair (1988); Shaw and Levin (2011); Wingfield (2003), which depend on how, when and where the animals perform the movements. Therefore, this diffusion drives to the evolution of the migration patterns Fryxell and Sinclair (1988); Griswold et al. (2010); Watts et al. (2018).

From the perspective of migration biology, migratory patterns can be grou-ped in four types, namely obligated, nomadic, partial and fugitive migration Chapman et al. (2011); Dingle and Drake (2007); Newton (2012, 2008); Roshier et al. (2006); Watts et al. (2018); Wingfield (2003). However, this classification is a simplification because these migratory types are not mutually exclusive Dingle and Drake (2007); Watts et al. (2018). Thus, nomadic and fugitive migrations are also called facultative migrations and therefore are migratory phenomena that respond to random fluctuations in the resources availability with which are unpredictably affected in space and/or time Boyle (2017); Chapman et al. (2011); Cornelius et al. (2013a); Griswold et al. (2010); Kokko (2011); Swingland et al. (1983). The focus of this paper is the obligate migration, also known as calendar migrants, in this type of migration all individuals within the population make regular annual trips between the wintering and breeding areas, which may include stopovers to rest and / or move. The patterns of movement in this migration are associated with the distribution of resources, so the position of the population can be predicted temporarily Chapman et al. (2011); Newton (2012); Watts et al. (2018).

One of the basic principles stimulating the animal migration is the variability of the resources availability both in time and space, which allow to predict when and where species will move Chapman et al. (2011); Cornelius et al. (2013b); Stojanovic et al. (2015); Watts et al. (2018). Particularly, species that perform forced (obligate) migrations are characterized to travel consistently in time, space, distance and movement direction from one site to another to exploit resources sufficiently predictable Cornelius et al. (2013a); Chapman et al. (2011); Cornelius et al. (2013b); Griswold et al. (2010); Kokko (2011). In this kind of migration, populations present a periodic or regular movement pattern in relation with the pursuit of seasonally predictable resources Kokko (2011); Swingland et al. (1983); Wingfield (2003), which permit to development all stages of their life cycle. Thus, the places to which the populations move are commonly breeding or hibernation sites Lande et al. (2017). In these cases, the location of the species is represented by a function that depends on the time that the species remains in a place to develop some stages in its life cycle and the speed at which it performs the migratory movement. In addition, residence place influences both the intrinsic growth rate, and the load capacity or medium support, Arditi et al. (2016); Carlos and Braumann (2017); Singh and Leonardsson (2014). Thus, the study of population dynamics in migrating species through logistic model with intrinsic growth rate and load capacity depending of the position will show a new and more real scenario to know the migration dynamics of populations.

The development of this paper is presented in six sections, in the first one, a description is made of the mathematical model that describes the population dynamics in which the intrinsic rate of growth and the carrying capacity depend on the position function; second, it shows how the population abundance is described by means of a discrete mapping, besides these the fundamental work theorem is presented; third and fourth, illustrates the sensitivity of population abundance with respect to the parameters of the model; fifth, the discussion centered on the theorem described in session three and sixth, the appendix, where the lemmas and their demonstrations that serve as theoretical support to the development of the article are presented.

MATHEMATICAL MODEL

The population obligate-migration model

The logistic model is a classic to represent increase in the population abundance Arditi et al. (2016, 2018); Carlos and Braumann (2017), when there is dependence on the density in the per capita rate of growth. This assumes that this rate is a linearly decreasing function of the population density. Thus, considering a habitat of constant size, the logistic model assumes constant intrinsic growth rate r and carrying capacity K, where $\lambda = r/K$ is the growth loss by the unit increment of the population size, which is a measurement of the intensity of intraspecific competition.

If we consider a population that develops its activities in a one-dimensional macro area $I = [-a, +a]$, $a > 0$, also we will assume that the population location is visualized (at a convenient scale) as a point in the I interval and therefore, the location at each instant is given by periodic functions (*i.e.* of period the unit of time) $\phi_{\tau,\nu} : \mathbf{R} \to I$, so that the position of the population at time t is $\phi_{\tau,\nu}(t)$, where τ is a parameter associated with the immobility time and ν is one related to the displacement speed.

Assuming that the population abundance follows a logistic (r, K)-model , $K = r/\lambda$, we have that at certain time $t \in \mathbf{R}$ and place $x \in I$, the intrinsic growth rate is $r[t, x]$, this is, it depends functionally of the location x at that instant t. That is to say,

$$r = r[t, \phi_{\tau,\nu}(t)] \tag{1}$$

then, by denoting by $N(t)$ the abundance of the population at time t, and assuming that the intensity of instraspecific competition λ remains constant, we have the following abundance model:

$$N'(t) = r[t, \phi_{\tau,\nu}(t)]\, N(t) \left\{ 1 - \lambda \frac{N(t)}{r[t, \phi_{\tau,\nu}(t)]} \right\} \tag{2}$$

From (2), integrating $N(\cdot)/N'(\cdot)$ over the interval $[0, s]$, for a posterior convenient second integration for $s \in [0, t]$, we get:

$$N(t) = \frac{N_0 \, \exp\left(\int_0^t r[s, \phi_{\tau,\nu}(s)]\, ds\right)}{1 + \lambda\, N_0 \int_0^t \exp\left(\int_0^s r[\mu, \phi_{\tau,\nu}(\mu)]\, d\mu\right) ds} \tag{3}$$

Intrinsic growth rate: We will consider that given a position in the space $x \in I$, the intrinsic growth rate as a function of time, denoted by $r[t, x]$, follows a cyclical behavior, a periodic pattern, since the conditions (resources) in this place x are regulated by the annual seasonality. In addition, we will assume that if at one end of the space $[-a, a]$ the conditions are favorable, they are not towards the other one, and vice versa, when in one edge the conditions get worse, towards the other one they improve. The case to consider for $(t, x) \in [0, 1] \times I$, by simplicity is:

$$r[t, x] = \frac{1}{2} \left\{ [r(t, +a) - r(t, -a)] \frac{x}{a} + [r(t, +a) + r(t, -a)] \right\}, \tag{4}$$

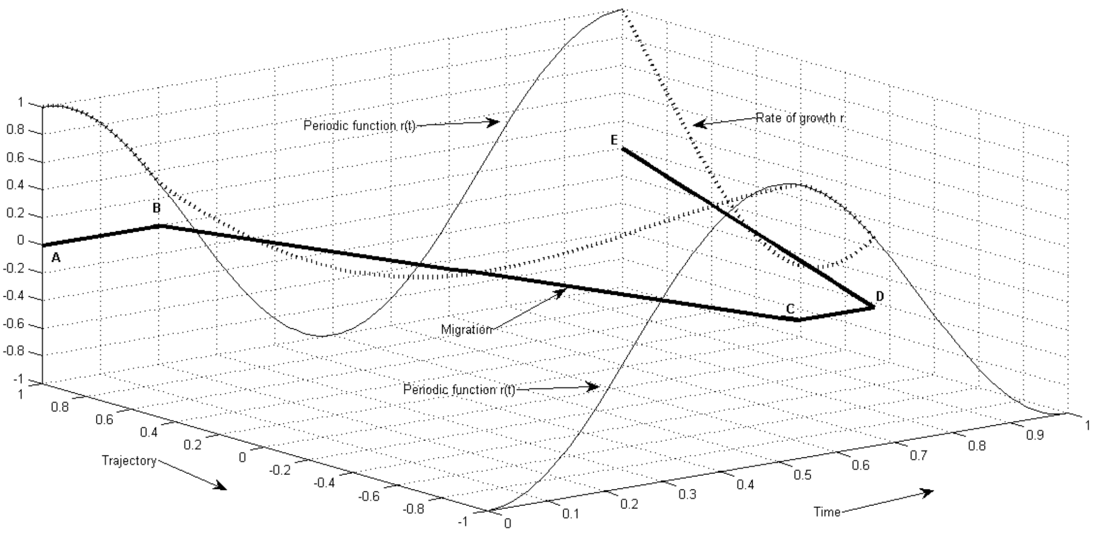


Figure 1: The intrinsec rate of growth plot as a time-state function. The migratory rute is described in dark straight lines. The axis corresponds to trajectory and time. The parameters are fixed as $t = [0,1]$, $\lambda = 1$, $r_\ominus = r_\oplus = 1$,$-a = +a = 1$, $\tau = 0.2$ and $\nu = 6$.

where $r(t,+a) = r_\oplus \cos(2\pi t)$ and $r(t,-a) = -r_\ominus \cos(2\pi t)$, with some positive constants $r_\oplus$ and $r_\ominus$. The graph of the function $r[t,x]$ is the wavelike and rectified surface that is visualized in Fig.1.

Population trajectory: The idea, in terms of trajectory to model, is that the population will remain in a place with the highest rate of favorable growth (*v.g.* positive) at one end of the space (*v.g.* $x = a$), but by seasonality, it will vary to an unfavorable one (*v.g.* negative) and if the population remains in such a place it will be affected. Then there will be a time when it should migrate looking for a positive rate. If it gets to this place, the other end of the space (*v.g.* $x = -a$),where it gets a positive rate again, it will remain there for another time. But at some point it will also vary negatively and it will return to the original end, completing the cycle (a unit of time). Assuming that the population moves at a constant velocity ν, $\nu > 0$, the total time in transit is $4a/\nu$. Therefore, the time in stillness at the ends is $1 - 2a/\nu$. Suppose that it is divided between two periods $\tau_\oplus$ and $\tau_\ominus$ depending on how much time it is respectively in $x = +a$ o $x = -a$. The population cycle is given four steps: remains in $x = a$ and migrate to $x = -a$, stay in $x = -a$ and return $x = a$. Total time is shown in the following equation:

$$\tau_\oplus + \tau_* + \tau_\ominus + \tau_* = 1, \qquad (5)$$

in which $\tau_* := 2a/\nu < 1/2$, is assumed, so that the population has the possibility of completing the cycle. In order to reduce the parameters in view, we will denote $\tau_\oplus = \tau$ and $\tau_\ominus = 1 - (\tau + 2\tau_*)$, subject to:

$$\tau < 1 - 2\tau_* \quad \text{and} \quad \tau_* < 1/2 \qquad (6)$$

In this way, the function $\phi_{\tau,\nu} : \mathbf{R} \to [-a,a]$ of locating the population in space-time is:

$$\phi_{\tau,\nu}(t) = \begin{cases} a & \text{if} \quad t \in [0,\tau], \\ a - \nu(t-\tau) & \text{if} \quad t \in [\tau, \tau+\tau_*], \\ -a & \text{if} \quad t \in [\tau+\tau_*, 1-\tau_*], \\ a + \nu(t-1) & \text{if} \quad t \in [1-\tau_*, 1] \end{cases} \qquad (7)$$

Note that $\phi'_{\tau,\nu}(t) = 0$ or $\phi'_{\tau,\nu}(t) = \pm\nu$, it changes sign, positive or negative, depending on whether the path is with origin or destination $x = -a$. See the pieceswise linear graph in Fig.1.

TABLE 1: VARIABLES AND PARAMETERS.

Concept		Concept	
Time	t	Stay time in $x = a$	τ
Population abundance	$N(\cdot)$	Velocyti of displacement	ν
Initial abundance	N_0	Territory's left edge.	$-a$
Intrinsic rate of growth	$r[\cdot]$	Territory's right edge.	$+a$
Intra-competition factor	λ	Amplitude of r in $x = -a$	$r_\ominus$
Position in the space	$x()$	Amplitude of r in $x = +a$	$r_\oplus$
Location function	$\phi_{\tau,\nu}(\cdot)$		

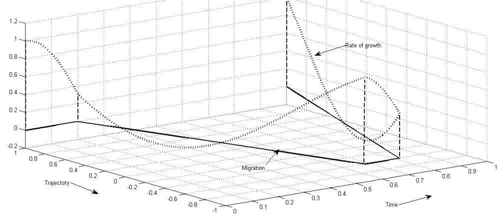
ANALYSIS AND RESULTS

The abundance equation (3) considers the term $r[t,x]$, with $x = \phi_{\tau,\nu}(s)$, so that

$$r[s,\phi_{\tau,\nu}(s)] = \frac{1}{2}\cos(2\pi s)\left\{[r_\oplus + r_\ominus]\frac{\phi_{\tau,\nu}(s)}{a} + [r_\oplus - r_\ominus]\right\} \qquad (8)$$

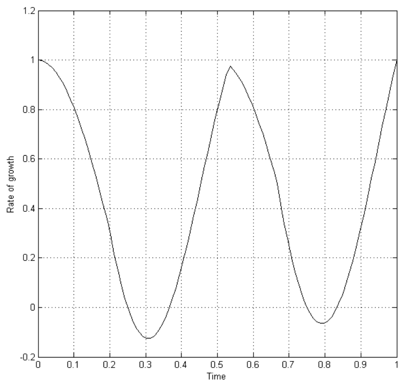
where $\phi_{\tau,\nu}(\cdot)$ is given by (7). The graph of $r[t,\phi_{\tau,\nu}(t)]$, $t \in [0,1]$ is shown in Fig. **??** ($r[\cdot]$ vs. $\phi_{\tau,\nu}$) and Fig. **??** ($r[\cdot]$ vs. t).

In order to know the dynamic behavior of the long-term population size, discrete mapping will be studied $\mathscr{M}_k : [0,\infty) \to [0,\infty)$ that relates the abundances between two terms of consecutive cycles. Note that there is a one-to-one correspondence between fixed points of $\mathscr{M}_k$ and 1-periodic trajectories of the abundance equation (2).



(a) r vs. $\phi_{\tau,\nu}$

Figure 2:



(a) r vs. t

Figure 3: Rate of growth according to (a) trajectory $\phi_{\tau,\nu}$. The axis corresponds to trajectory and time. In (b) the axis corresponds to rate of growth and time. In both graphs the parameters are fixed, as $t=[0,1]$, $\lambda=1$, $r_{\ominus}=r_{\oplus}=1, -a=+a=1$, $\tau=0.2$ and $\nu=6$.

About the definition of $\mathscr{M}_k(\cdot)$, note that denoting $N_j = N(j)$, $j \geq 1$, we have from (2) the $N_{k+1} = \mathscr{M}_k(N_k)$ relation, where $\mathscr{M}_k : [0,\infty) \to [0,\infty)$ is defined by:

$$\mathscr{M}_k(N) = \frac{N\,\eta(k,k+1)}{1+\lambda N \int_k^{k+1} \eta(k,s)\,ds}, \tag{9}$$

with $\eta(s,t) = \exp\left(\int_s^t r[u,\phi_{\tau,\nu}(u)]\,du\right)$

Consequently, using (9), Lemma 1 and Lemma 2 in the Appendix, we have that the map $\mathscr{M}_k(\cdot)$, relating the abundance at the beginning of two consecutive cycles, is autonomous of the time variable k. Then:

$$N_{k+1} = \mathscr{M}(N_k) = \frac{\eta(0,1)}{1+\lambda \mathscr{D}\, N_k} N_k \tag{10}$$

where $\mathscr{D} = \int_{[0,1]} \eta(0,u)\,du$ and

$$\eta(0,1) = \exp\left\{\nu \frac{r_{\oplus}+r_{\ominus}}{8\pi^2 a} \mathscr{A}\left(2\pi\tau_*, 2\pi\tau\right)\right\} \tag{11}$$

with $\mathscr{A}(u,v) = [1-\cos(u)][1+\cos(v)] + \sin(v)\sin(u)$, for $k \geq 0$.

So that, by simple successive replacements of (10) on itself, the formation of a geometric progression, and proving by induction (over time parameter k), it is clear that its solution is given by:

(a) If $\eta(0,1) \neq 1$,

$$N_k = \frac{\eta(0,1)^k}{1+\lambda \mathscr{D} N_0 \frac{\eta(0,1)^k - 1}{\eta(0,1)-1}} N_0, \quad k \geq 0 \tag{12}$$

where N_0 is the initial abundance.

(b) If $\eta(0,1) = 1$,

$$N_k = \frac{1}{1+k\lambda \mathscr{D} N_0} N_0, \quad k \geq 0 \tag{13}$$

Notice that the future behavior of sequence $\{N_k\}$ depends strongly on whether the value of $\eta(0,1)$ is greater than, less than or equal to one.

Theorem 1 *Let us consider (10). Apart from the zero abundance equilibrium, the unique positive one, that exists when* $\eta(0,1) > 1$*, is*

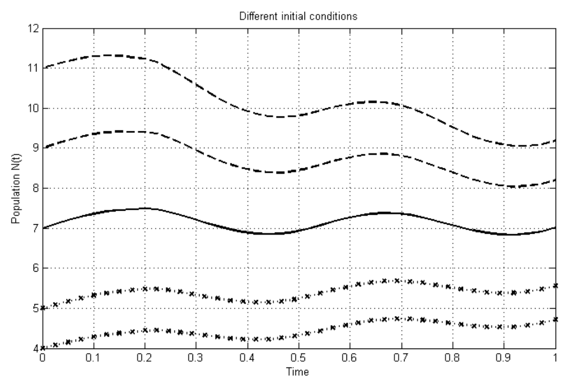
$$N_\infty := \frac{1}{\lambda \mathscr{D}} \{\eta(0,1) - 1\}$$

In addition, the asymptotic scenarios are:

(1) If $\tau < 1/2$ *(i.e.,* $\eta(0,1) > 1$*), then monotonously* $N_k \to N_\infty$ *as* $k \to \infty$*. This is, abundance defined by (2) tends to follow an oscillatory behavior (with period one) globally asymptotically stable. See Fig.4a, Fig.5a and Fig.5b.*

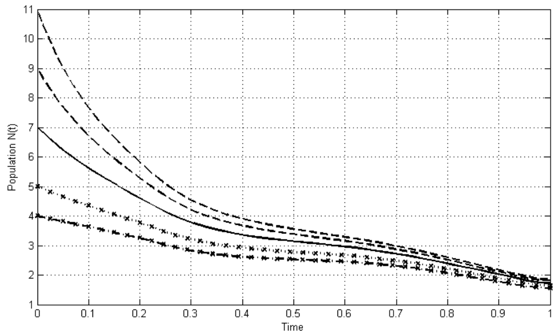
(2) If $1/2 < \tau < 1 - 2\tau_*$ *(i.e.,* $\eta(0,1) \leq 1$*), then monotonously* $N_k \to 0$ *as* $k \to \infty$*. Therefore, the population necessarily goes to extinction at future time. See Fig. 4a.*

Proof: If $\eta(0,1) > 1$, assertion (1) follows immediately from (12) dividing by $\eta(0,1)^k$ and taking the limit as $k \to \infty$. When $\eta(0,1) < 1$, we have to take the limit directly for getting (2). The case (3) follows if $\eta(0,1) = 1$. In addition, for knowing the sign of $\eta(0,1) - 1$, by (11) it is necessary the study of signs of function $A(2\pi\tau_*, 2\pi\tau)$ in terms of the parameters τ and τ_*. See Lemma 3 in Appendix. $\diamond$

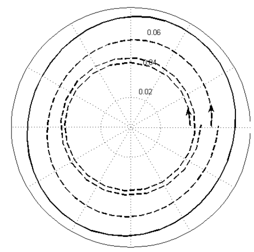


(a) $\tau < 1/2$

Remark 1: Notice that τ is the time that remains on the edge a and $\tau_* = 2a/\nu$ is the time it takes the population to reach the other end of the habitat. Therefore, according the Theorem 1, the population persists only when $\tau < 1/2$ and $\tau < 1 - 2\tau_*$. Thus, there is a minimal habitat crossing speed to overcome, a kind of "escape speed", which equal to $2a/\tau$.



(a) $1/2 < \tau < 1 - 2\tau_*$



(a) $\tau < 1/2$

Figure 4: Population dynamics during the first cycle with five initial conditions: $N_{0_1} = 4$, $N_{0_2} = 5$, $N_{0_3} = 7$, $N_{0_4} = 9$, $N_{0_5} = 11$. In both the vertical axis corresponds to population abundance $N(t)$ and horizontal axis corresponds to time t. Some parameters are fixed, as $\lambda = 1$, $r_\ominus = r_\oplus = 1, -a = +a = 1$. Population dynamics: in (a) parameters satisfy assertion (1) of Theorem 1 with $\tau = 0.2$ and $\nu = 6$. In (b) parameters satisfy assertion (2) of Theorem 1 with $\tau = 0.9$.

Figure 5: Population dynamics during the first cycle with different initial conditions, the full angle corresponds to a unit of time and radial length to abundance. In both some parameters are fixed, as $\lambda = 1$, $r_\ominus = r_\oplus = 1, -a = +a = 1$. In (a) there are three initial conditions as $N_{0_1} = 4$, $N_{0_2} = 5$, $N_{0_3} = 7$. In (b) there are three initial conditions as $N_{0_4} = 9$, $N_{0_5} = 11$, $N_{0_3} = 7$, besides this parameters satisfy Theorem 1 with $\tau = 0.2$ and $\nu = 6$, so population dynamics tends to the stable state.

SENSIBILITY OF ABUNDANCE TO THE PARAMETERS.

Average abundance N_∞

The Theorem 1, when $\eta(0,1) > 1$, shows an abundance of stable equilibrium N_∞, which determines a periodic trajectory, given by the initial condition:

$$N_\infty(r_\oplus, r_\ominus, \lambda, a, \tau, \nu) = \frac{\eta(0,1) - 1}{\lambda \int_0^1 \eta(0,z)\,dz} \qquad (14)$$

Remember that

$$\eta(0,1) = \exp\left\{ \nu \frac{r_\oplus + r_\ominus}{8\pi^2 a} \mathscr{A}\left(2\pi\tau_*, 2\pi\tau\right) \right\} \qquad (15)$$

with $\mathscr{A}(u,v) = [1 - \cos(u)][1 + \cos(v)] + \sin(v)\sin(u)$, for $k \geq 0$. It shows the different scenarios of the population dynamics N_∞ to changes in the parameters as shown in Fig.6 and Fig.7:

Population abundance vs. parameters: τ, ν *and* $r_\ominus$

According to the definition (7), the parameter τ indicates the permanence at the edge $\phi_{\tau,\nu}(t) = a$ for $t \in [0,\tau]$. At that place, at time $t = 0$, it is in the rate of maximum positive growth $r_\oplus$, this will go down to become negative at $t = 1/4$. There is a great variability regarding how the moment (τ), in which that location is abandoned, influences the dynamics during a cycle $t \in [0,1]$, since in $t = 1$ it returns, as shown in Fig. **??**. In general, it is observed that if choosing the time of permanence in the initial position well, the better it will be to chase of the high values of r. Note that a late abandonment could mean a clear risk of extinction, as shown in Fig.6. It should be noted that when the population remains in the same place during the whole period of time $(\tau = 1)$, the conditions are not favorable in the place and the population tends to zero slowly, as shown by the dotted line of Fig.6.

Population dynamics are being affected by different parameters that modify their evolution. The parameter τ refers to

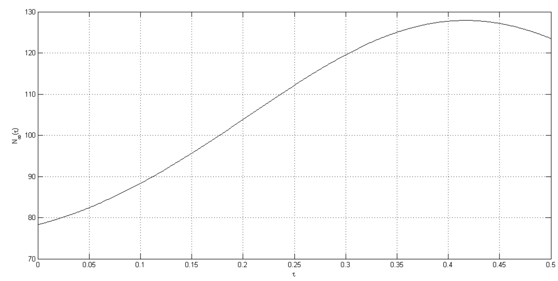


Figure 6: Populatation dynamic $N_\infty(r_\oplus, r_\ominus, \lambda, a, \tau, \nu)$ vs. parameters τ. Vertical axis corresponds to population abundance $N_\infty(\cdot)$ and horizontal axis corresponds to parameter τ. Some parameters are fixed, as $\lambda = 1$, $r_\ominus = r_\oplus = 1, -a = +a = 1$. In Population dynamics, parameters satisfy assertion (1) of Theorem 1, $\tau < 1/2$ for all values of τ.

the residence time of the population at a certain stage before starting its journey at a speed ν. The graphs show the population dynamics through time in different scenarios generated by the change in the parameters.

There is another parameter $r_\ominus$ that represents Left amplitude in the growth rate presenting the population dynamics. As shown in Fig.9, this parameter does not affect the behavior in the abundance dynamics of the population, but it helps to improve its quantity, so that the greater the amplitude, the better the abundance.

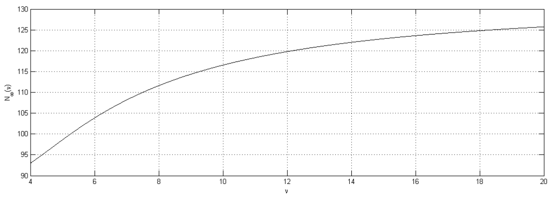
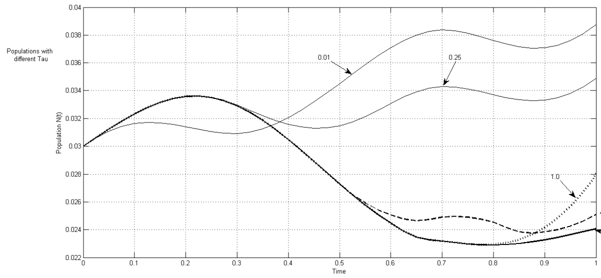
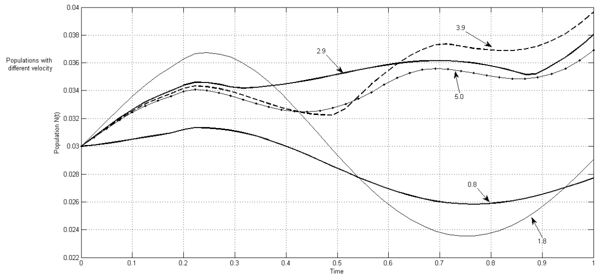


Figure 7: Populatation dynamic $N_\infty(r_\oplus, r_\ominus, \lambda, a, \tau, \nu)$ vs. parameters ν. Vertical axis corresponds to population abundance $N_\infty(\cdot)$ and horizontal axis corresponds to paramete ν. Some parameters are fixed, as $\lambda = 1$, $r_\ominus = r_\oplus = 1, -a = +a = 1$. In Population dynamics, parameters satisfy assertion (1) of Theorem 1, $\tau < 1/2$ for all values of ν and $\tau < 1 - 2\tau_*$.



(a) Population-τ



(a) Population-ν

Figure 8: Sensitivity to parameters τ and ν. In both the vertical axis corresponds to $N(t)$ (population abundance) and horizontal axis corresponds to time. Some parameters are fixed, as $t = [0, 1]$, $\lambda = 1$, $r_\ominus = r_\oplus = 1, -a = +a = 1$ and $\nu = 50$. There are five conditions for τ denoted by τ_i as $\tau_1 = 0.01$, $\tau_2 = 0.25$, $\tau_3 = 0.501$, $\tau_4 = 0.75$, $\tau_5 = 0.92$ in (a) for population dynamics, in the other hand in (b) $\tau = 0.2$ and there are five conditions for ν denoted by ν_i as $\nu_1 = 0.8$, $\nu_2 = 1.8$, $\nu_3 = 2.9$, $\nu_4 = 3.9$, $\nu_5 = 5.0$.

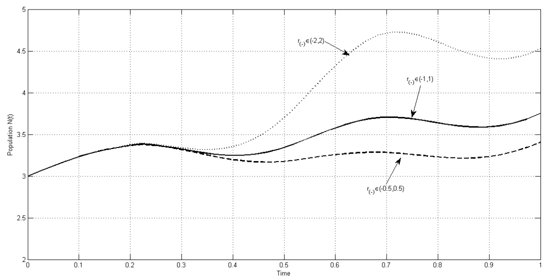


Figure 9: Sensitivity to parameter $r_\ominus$. The vertical axis corresponds to $N(t)$ (population abundance) and horizontal axis corresponds to time. Some parameters are fixed, as $t = [0, 1]$, $\lambda = 1$, $r_\oplus = 1, -a = +a = 1$, $\tau = 0.2$ and $\nu = 6$. There are three conditions for $r_\ominus$ denoted by $r_{i\ominus}$ as $r_{1\ominus=0.5}$, $r_{2\ominus} = 1.0$, $r_{3\ominus} = 2.0$ for different population dynamics.

DISCUSSION

General dynamics of the abundances in the scenario presents an oscillating behavior with an upward trend or a decrease that stabilizes in the long term in an oscillatory regime (periodic) in a calendar year, as the Fig. 6a y Fig. 6b. show. Regardless of the initial condition, there is a tendency to a periodic defined by the parameters considered, which are four:(a) those associated with the trajectory τ and ν, and (b) Those related to the growth rate $r_\oplus$ and $r_\ominus$, except for the spatial competition a and competition inter-specific λ. Particularly important for the population are those defined in (a), since the species depend on them and are related to adaptation, which in the long term can be altered by evolutionary factors. As for τ and ν, as determinants of the dynamics, in the short term, it can be observed that when passing a cycle one has:

- Note that according to (5) τ is the dwell time of the population at the far right of the space $[-a, a]$. In this extreme, the growth rate is $r[s, \phi_{\tau,\nu}(s)] = r_+ \cos(2\pi s)$ to $s \in [0, \tau]$, that is, τ is the residence time from the moment in which the p.r.g. It presents its highest value and begins to decline. Fig. 3 (a) shows that, compared to the abundance after a cycle, if the population leaves the site quickly, it presents a more favorable environment than that which would have had to wait a while at the site. On the other hand, if the population delays the place late, it can be worse than choosing to stay in the place.
- The function $\phi_{\tau,\nu}(\cdot)$ defined in (5) indicates that ν represents the speed of displacement from one end of the space to another $[-a, a]$. Fig. 3. shows that in principle high speeds allow a better yield in abundance, taking advantage of the best growth rates provided that $\tau < 1/2$, otherwise, the abundance of the population tends to zero.

Assuming that the conditions of the species are favorable and can meet the assumptions so that the abundance of its population converges to an oscillating but stationary regime, defined by an initial condition of abundance that after a cycle is the same, a natural question is what is the combination of these traits that optimize this initial condition so that it defines the final seasonality ?, example of this is the average population shown in Fig. 5 and Fig. 6. In both cases the population abundance reaches stability condition.

APPENDIX

Lemmas and their demonstrations are presented in which the calculations are supported.

Lemma 1 *Considering the periodicity of the function $\phi_{\tau,\nu}(\cdot)$ and that of the components of $r[\cdot, x]$, we have the following equality*

$$\int_k^{k+1} r[s, \phi_{\tau,\nu}(s)]\, ds = \nu \frac{r_\oplus + r_\ominus}{8\pi^2 a} \mathscr{A}\left(2\pi\tau_*, 2\pi\tau\right) \qquad (16)$$

where $\mathscr{A}(u,v) = [1-\cos(u)][1+\cos(v)] + \sin(v)\sin(u)$*, for* $k \geq 0$.

Proof: From the definitions (8) and (7), making the necessary composition it is obtained (8). Denoting the integral of $r[\cdot, \phi_{\tau,\nu}(\cdot)]$ on $[k, k+1]$ by $\mathscr{J}_r(k)$, $k \geq 0$, the periodicity of the functions involved implies:

$$\mathscr{J}_r(k) = \mathscr{J}_r(0) = \frac{r_\oplus + r_\ominus}{2a}\int_0^1 \phi_{\tau,\nu}(s)\cos(2\pi s)\,ds + \frac{r_\oplus - r_\ominus}{2}\int_0^1 \cos(2\pi s)ds \quad (17)$$

Since the last integral is zero, by integrating the first term to the right of equality by parts, this value is reduced to

$$\mathscr{J}_r(0) = -\frac{r_\oplus + r_\ominus}{4\pi a}\int_0^1 \phi'_{\tau,\nu}(s)\sin(2\pi s)\,ds$$

Considering that $\phi'_{\tau,\nu}(\cdot) = 0$ over the intervals $[0,\tau]$ and $[\tau + \tau_*, 1-\tau_*]$, we have

$$\mathscr{J}_r(0) = -\frac{r_\oplus + r_\ominus}{4\pi a}\left\{-\nu\int_\tau^{\tau+\tau_*}\sin(2\pi s)ds + \nu\int_{1-\tau_*}^1 \sin(2\pi s)ds\right\} \quad (18)$$

Thus, solving the integrals

$$\mathscr{J}_r(0) = \nu\frac{r_\oplus + r_\ominus}{8\pi^2 a}\left\{\cos(2\pi s)|_{1-\tau_*}^1 - \cos(2\pi s)|_\tau^{\tau+\tau_*}\right\}$$

As the trigonometric expression between braces, after the cosine decomposition of a sum, it is equal to $[1-\cos(2\pi\tau_*)][1+\cos(2\pi\tau)] + \sin(2\pi\tau)\sin(2\pi\tau_*)$. It follows the expression (16). ⋄

Lemma 2 *The periodicity of the function* $\phi_{\tau,\nu}(\cdot)$ *and that of the components of* $r[\cdot,x]$*, imply* $\eta(k,k+1) = \eta(0,1)$ *and*

$$\mathscr{D} := \int_k^{k+1}\eta(k,s)\,ds = \int_0^1 \eta(0,u))du \quad (19)$$

for $k \geq 1$*. that is,* $\mathscr{M}_k$ *defined by (9) does not depend on k,* $k \geq 1$.

Proof: This is immediate from (9) and (16). ⋄

Lemma 3 *Note that, with the notation* $\tau_* = 2a/\nu$, $u = 2\pi\tau_*$ *and* $v = 2\pi\tau$*, we have:*

(A) If $\tau \in]0,1/2[$*, then* $\mathscr{A}(u,v) > 0$.

(B) If $\tau \in]\frac{1}{2}, 1-2\tau_*[$*, then* $\mathscr{A}(u,v) < 0$.

Proof: Since $\mathscr{A}(u,v) = [1-\cos(u)][1+\cos(v)] + \sin(v)\sin(u)$, by trigonometric identities (expressing $\sin(u-v)$ in terms of sine and factoring de difference $\cos(v) - \cos(u)$), we have that $\mathscr{A}(u,v) = 4\sin[(u+v)/2]\sin[u/2]\cos[v/2]$. that is,

$$\mathscr{A}(2\pi\tau_*, 2\pi\tau) = 4\sin[\pi(\tau_* + \tau)]\sin[\pi\tau_*]\cos[\pi\tau] \quad (20)$$

Since $\tau_* < 1/2$, we have $0 < \pi\tau_* < \pi/2$, then $\sin(\pi\tau_*) > 0$. Therefore, we have

$$sgn\{\mathscr{A}(2\pi\tau_*, 2\pi\tau)\} = sgn\{\sin[\pi(\tau_* + \tau)]\cos[\pi\tau]\}$$

Two cases:

(A) If $\tau_* < 1/2$ and $0 < \pi\tau_* < \pi/2$ then $\tau < 1/2$ and $\pi\tau < \pi/2$ so $\cos[\pi\tau] > 0$ now $\sin[\pi(\tau + \tau_*)] = \sin[\pi\tau + \pi\tau_*] > 0$. So that, $sgn\{\mathscr{A}(2\pi\tau_*, 2\pi\tau)\} > 0$.

(B) if $\tau_* > 1/2$ then $\pi > \pi\tau > \pi/2$ so $\cos[\pi\tau] < 0$ now $-\sin[\pi(\tau + \tau_*)] = \sin[\pi\tau + \pi\tau_*]$ where $\pi\tau \in [\pi/2, \pi]$ and $\pi\tau_* \in [0, \pi/2]$ so $\sin[\pi\tau + \pi\tau_*] < 0$ So that, $sgn\{\mathscr{A}(2\pi\tau_*, 2\pi\tau)\} < 0$.

Thus the proof is concluded. ⋄

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Estudio de un modelo predador-presa con tres especies y capacidad de carga variable

Study of a predator-prey model with three species and variable carrying capacity

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Resumen— Esta investigación se centró en el estudio de un ecosistema en el que coexisten dos depredadores, *Puma concolor* y *Lycalopex culpaeus*, los cuales compiten por presas en común. En este modelo también se considera que el puma depreda al Zorro culpeo, lo que agrega una dinámica adicional al sistema. Se utilizaron ecuaciones diferenciales para modelar la interacción entre estas tres especies, considerando la capacidad de carga variable de las presas y suponiendo que no hay migración ni depredación externa. Se realizaron 256 simulaciones utilizando diferentes valores de parámetros, y se encontraron diversos escenarios donde se produjo la extinción de ambas especies depredadoras, la extinción exclusiva del Puma o del Zorro culpeo, y situaciones en las que el crecimiento excesivo de un depredador llevó a la disminución de la población de presas y, por lo tanto, a la extinción del sistema completo. Los resultados mostraron que la tasa de consumo de los predadores es uno de los factores que más regula el comportamiento del sistema. También se observó que el cambio en la tendencia de la capacidad de carga afectó las densidades máximas de presas que se pudieron lograr.

Palabras clave—Predador-presa, Respuesta Funcional, Capacidad de carga, Simulaciones numéricas, *Puma concolor*, *Lycalopex culpaeus*

Abstract—This research focused on the study of an ecosystem in which two predators, *Puma concolor* and *Lycalopex culpaeus*, coexist and compete for common prey. This model also includes that the Puma preys on the Culpeo fox, which adds dynamic to the system. Differential equations were used to model the interaction between these three species, considering the variable carrying capacity of prey and assuming no migration or external predation. 256 simulations were performed using different parameter values, and various scenarios were found where both predator species became extinct, exclusive extinction of either the Puma or the Culpeo fox occurred, and situations where the excessive growth of a predator led to a decrease in prey population and therefore extinction of the entire system. The results showed that the consumption rate of predators is one of the factors that most regulate the system's behavior. It was also observed that the change in the trend of carrying capacity affected the maximum densities of prey that could be achieved.

Keywords—Prey-predator, Functional response, Carrying capacity, Numerical simulations, *Puma concolor*, *Lycalopex culpaeus*

Introducción

La competencia interespecifica se refiere a la interacción entre individuos de diferentes especies que compiten por los mismos recursos limitados en un mismo hábitat o ecosistema (Kitzes, 2022). El modelamiento matemático de competencia entre especies tiene sus orígenes en el modelo de Lotka–Volterra desarrollado por la década de 1920.

Es posible encontrar una serie de publicaciones científicas y estudios recientes relacionados a la temática de modelos predador-presa, donde destacan temas como el análisis de los patrones temporales en función de distintas respuestas funcionales (Majumdar et al., 2022; Naik et al., 2022a; Jana y Kumar Roy, 2022; Barman y Ghosh, 2022) y el análisis del "Efecto Alle" y del "Efecto miedo" en las presas (Li et al., 2022; Naik et al., 2022b; Devi y Jana, 2022; Lan et al., 2022; Gökçe, 2022).

Otro tipo de estudios han modelado la interacción de más de dos especies en un ecosistema. En el estudio de Pang y Wang (2004) se analizaron las características de un ecosistema formado por dos predadores que compiten por la misma presa. Por otro lado, también se han investigado modelos que incluyen la cooperación entre dos presas para protegerse de un predador en común (Kundu y Maitra, 2018).

Capacidad de carga variable

El concepto de capacidad de carga de un ecosistema se refiere a la cantidad máxima de individuos de una especie que puede sostener un ecosistema o un hábitat determinado de manera sostenible a largo plazo, sin agotar los recursos naturales y sin causar daños irreparables al medio ambiente. En el estudio de Al-Moqbali et al. (2018) se propone un modelo predador- presa que incorpora una ecuación que describe los cambios en el tiempo de la capacidad de carga del ecosistema. Esta se define como:

$$\frac{dk}{dt} = \alpha(k(t) - k_1)\left(1 - \frac{k(t) - k_1}{K_2}\right), \qquad (1)$$

donde k es la capacidad de carga que aumenta sigmoidalmente entre un valor inicial k_0 mayor a k_1 y un valor final $k_1 + k_2$ con una tasa de crecimiento α.

Respuestas Funcionales

La respuesta funcional es la relación entre la tasa de depredación (Presas/tiempo) y la densidad de presas (Smith y Smith, 2007). Holling (1965) tipificó las respuestas funcionales en tres clases diferentes:

Tipo I: Esta respuesta funcional se basa en el supuesto de que el cambio en la densidad de la población de predadores es proporcional a la densidad de la población de presas disponible (x). La expresión matemática asociada a esta respuesta corresponde a

$$h(x) = \begin{cases} \gamma x & \text{si} \quad 0 \leq x < c \\ \\ \gamma c & \text{si} \quad c > x, \end{cases}$$

donde x es la densidad de presas. Se puede entender que existe un aumento lineal de consumo de los predadores respecto a la densidad de población de presas, llegando a un punto donde este valor es constante (Ej: fitoplancton y zooplancton).

Tipo II: En este tipo de respuesta el número de presas consumidas por el predador se incrementa pero con una tasa decreciente, en función del incremento de la densidad de la presa (Badii et al., 2013). Es la llamada respuesta Monod de tipo hiperbólica, donde el parámetro γ es la tasa máxima de consumo percápita y a es la tasa de saturación media, es decir, la cantidad de presas en el que la tasa de depredación alcanza la mitad de su valor máximo (Garay-Gonzales, 2020). El modelo que describe este comportamiento es

$$h(x) = \frac{\gamma x}{a + x}.$$

Tipo III: Los predadores con este tipo de respuesta tienen una dieta basada en distintas especies de presas y su consumo es proporcional a sus abundancias, cambiando a las especies más abundantes y por tanto, permiten que las especies con menor densidad poblacional tengan oportunidad de incrementar sus poblaciones de nuevo (Badii et al., 2013). Es una respuesta sigmoidal que incluye la característica de que los predadores son ineficientes cuando los niveles de presas son bajos, y descrita por Al-Moqbali et al. (2018). Su ecuación matemática corresponde a

$$h(x) = \frac{\gamma x^2}{a^2 + x^2},$$

donde γ y a tienen el mismo sentido ecológico que en la respuesta funcional tipo II.

El Puma

El *Puma concolor*, también conocido como león de montaña o cougar, es una especie de felino nativo de América. En Chile, es uno de los grandes depredadores de los ecosistemas de montaña y es considerado un símbolo nacional de la conservación de la biodiversidad (Toledo y Surot, 2003). A pesar de su importancia ecológica, el *Puma concolor* enfrenta numerosas amenazas en Chile, incluyendo la caza ilegal, la pérdida de hábitat y la disminución de sus presas naturales (Ríos, 2009; Bonacic, 2013). La madurez sexual de las hembras se alcanza alrededor de los dos años, mientras que en los machos esta se da al final de los tres años y el periodo de gestación de las crías es de entre 90 y 95 días, teniendo camadas de entre dos y tres individuos cada dos años (Toledo y Surot, 2003). Estudios previos han estimado la densidad de pumas en distintas zonas del país, estando estas en un rango de entre 0,75 y 6 individuos por 100 kilómetros cuadrados (Guarda et al., 2017; Bonacic, 2013). Este carnívoro tiene dentro de sus presas más comunes alces, ciervos, venados, castores, ardillas, marmotas, lauchas, ratones, conejos, liebres y jabalíes, pero en Chile depreda particularmente lagomorfos (Conejos) y camélidos (Vicuñas y guanacos), aunque también hay evidencia de que consume otros carnívoros de menor tamaño como la especie *Lycalopex culpaeus*, mejor conocido como Zorro culpeo (Toledo y Surot, 2003; Pacheco et al., 2004).

Zorro culpeo

El zorro culpeo (*Lycalopex culpaeus*) es una especie de cánido que se encuentra ampliamente distribuida en el Neotrópico, habitando a lo largo de la cordillera de Los Andes desde Colombia hasta Tierra del Fuego (Zúñiga y Fuenzalida, 2016). Esta especie se describe comúnmente como una especie adaptable y generalista que puede adaptarse a cambios en su entorno tanto en términos de su alimentación como de su hábitat (Guntiñas Rosado et al., 2018). Se ha estudiado que en la zona centro-sur de Chile su dieta se compone de distintos tipos de roedores y lagomorfos, aunque en menor medida también consume aves y algunos reptiles de menor tamaño (Zúñiga y Fuenzalida, 2016). Los resultados de diversas investigaciones realizadas en Chile y Argentina, muestran que la densidad de zorros varía significativamente entre las distintas regiones estudiadas y oscila entre 0,2 y 2,6 individuos por kilómetro cuadrado, dependiendo del área y del método utilizado para su medición (MMA, 2020). Las hembras son monoéstricas, con un periodo de gestación de 65 días, mientras el número de crías varía entre 3 y 5 individuos que alcanzan la madurez sexual al año de edad

(Amster, 2009).

En el presente trabajo se estudian las características de un modelo matemático que representa un ecosistema conformado por un depredador dominante (Puma), un depredador intermedio (Zorro culpeo) y una presa, que tiene asociada una capacidad de carga variable y que asocia respuestas funcionales Holling Tipo II y III a los predadores.

EL MODELO MATEMÁTICO

Para este estudio se considera un ecosistema en donde existe un predador clava o dominante (Puma) y un predador secundario (Zorro culpeo), los cuales consumen a presas en común. En situaciones donde las presas sean escasas, el predador dominante puede consumir al predador secundario. Este comportamiento se puede regular asignando una respuesta funcional del tipo II al predador dominante y una respuesta funcional Tipo I al secundario. Para las presas se asocia un crecimiento de tipo logístico y la capacidad de carga será variable y se modelará con la ecuación 1. Así formulamos el siguiente modelo

$$\begin{cases} \dfrac{dx}{dt} = \alpha_1 \left(\frac{\gamma_1 z(t)^2}{a_1^2+z(t)^2}\right) x(t) + \alpha_1 \left(\frac{\gamma_2 y(t)^2}{a_2^2+y(t)^2}\right) x(t) - \beta_1 x(t) \\ \dfrac{dy}{dt} = \alpha_2 \left(\frac{\gamma_3 z(t)}{a_3+z(t)}\right) y(t) - \left(\frac{\gamma_2 y(t)^2}{a_2^2+y(t)^2}\right) x(t) - \beta_2 y(t) \\ \dfrac{dz}{dt} = \alpha_3 \left(1 - \frac{z(t)}{k(t)}\right) z(t) - \left(\frac{\gamma_1 z(t)^2}{a_1^2+z(t)^2}\right) x(t) - \left(\frac{\gamma_3 z(t)}{a_3+z(t)}\right) y(t) \\ \dfrac{dk}{dt} = \alpha_4 (k(t) - k_1) \left(1 - \frac{k(t)-k_1}{K_2}\right), \end{cases} \tag{2}$$

donde $x(t)$ es la densidad del Puma en un instante t, $y(t)$ es la densidad del Zorro culpeo en un instante t, $z(t)$ es la densidad de la presa en un instante t, $k(t)$ es la capacidad de carga en un instante t,α_1 es la tasa de reproducción del Puma, donde α_2 es la tasa de reproducción del Zorro culpeo, donde α_3 es la tasa de reproducción de la presa, α_4 es la tasa de crecimiento de la capacidad de carga, γ_1 es la tasa máxima de consumo percápita del Puma con la presa, γ_2 es la tasa máxima de consumo percápita del Puma con el Zorro culpeo, γ_3 es la tasa máxima de consumo percápita del Zorro culpeo con la presa, a_1 es la constante de saturación del Puma con la presa, a_2 es la constante de saturación del Puma con el Zorro culpeo, a_3 es la constante de saturación del Zorro culpeo con la presa, β_1 es la tasa de mortalidad del Puma , β_2 es la tasa de mortalidad del Zorro culpeo y $k_1 + k_2$ es el valor final de capacidad de carga.

Este modelo asume que las poblaciones interactúan entre sí y con el ambiente según las ecuaciones diferenciales que se presentan. También se supone que las tasas y parámetros son constantes en el tiempo y que no hay migración ni depredación externa.

Discusión acerca del modelo a priori

El sistema descrito en (2) representa un modelo que se ajusta a las características ecológicas del Puma y del Zorro culpeo, debido a que considera sus hábitos predatorios en su formulación. Al evaluar su credibilidad con base en los atributos propuestos por Berryman et al. (1995), se observa que el sistema (2) cumple en su mayoría con estos requisitos. Destacamos que el modelo, al incluir una ley dinámica para la capacidad de carga, considera un ambiente variable para las especies.

No obstante, sería recomendable personalizar aún más el modelo considerando variables adicionales, como el tamaño y periodo de gestación de cada especie, así como incluir factores en la ley dinámica de la capacidad de carga que aborden los efectos antropogénicos que afectan a las especies.

SIMULACIONES NUMÉRICAS

Utilizando los parámetros presentados en la Tabla 1 se realizaron 256 simulaciones para el modelo del sistema 2. Para resolver estas ecuaciones se utilizó la librería DeSolve del programa R (Soetaert et al., 2015) y las gráficas se implementaron con las librerías ggplot (Wickham et al., 2016) y ploty (Sievert, 2020). Los valores iniciales de las variables fueron $x(t) = 3$ individuos por 100 km^2, $y(t) = 3$ individuos por 100 km^2, $z(t) = 300$ individuos por 100 km^2 y $k(t) = 300$ individuos por 100 km^2. La unidad considerada para el tiempo fueron meses, trascurriendo 500 meses en las simulaciones. Las gráficas de las densidades de cada especie fueron realizadas en escala logarítmica ($log(x+1)$), debido a las diferencias en las magnitudes numéricas de las abundancias.

TABLA 1: PARÁMETROS UTILIZADOS EN LA SIMULACIONES DE MODELO 2

Parámetros para simulaciones	Definición	Valores
α_1	Tasa de reproducción Puma	0.1; 0.3
α_2	Tasa de reproducción Z. culpeo	0.3; 0.6
α_3	Tasa de reproducción presa	0.5
α_4	Tasa de crecimiento capacidad de carga	0.01
γ_1	Tasa máxima consumo percápita Puma-presa	0.4; 0.8
γ_2	Tasa máxima consumo percápita Puma-Z. culpeo	0.01; 0.03
γ_3	Tasa máxima consumo percápita Z. culpeo-presa	0.2; 0.8
a_1	Constante de saturación Puma-presa	0.4
a_2	Constante de saturación Puma-Z. Culpeo	0.9
a_3	Constante de saturación Z. Culpeo-presa	0.9
β_1	Tasa de mortalidad Puma	0.2; 0.3
β_2	Tasa de mortalidad Z. culpeo	0.05; 0.15
k_1	Parámetro capacidad carga	200
k_2	Parámetro capacidad carga	20; 150

La simulación de diferentes combinaciones de parámetros es un proceso computacionalmente costoso, y agregar nuevos valores incrementa exponencialmente la cantidad de ciclos que el programa debe ejecutar. Por esta razón se optó por variar únicamente aquellos parámetros que, luego de una inspección previa, se determinaron como los más significativos para generar diversos tipos de soluciones. Esta estrategia permitió reducir los tiempos de simulación sin afectar la calidad de los resultados obtenidos.

Entre los resultados arrojados por las simulaciones, destacan diversos escenarios donde se produce la extinción de ambos depredadores, la extinción exclusiva del Puma, así como también la extinción de Z. culpeo. Además, se observaron simulaciones en las cuales el crecimiento excesivo de un depredador resultó en la disminución de la población de presas, lo que a su vez llevó a la extinción del sistema en su totalidad. Se pudo apreciar el efecto de la capacidad de carga variable, analizando soluciones donde esta variable aumenta o disminuye. A continuación, se presentan los escenarios más relevantes generados en las simulaciones.

Extinción Predadores

Estás simulaciones se caracterizan por tener valores de los parámetros γ bajos, por lo que los predadores no alcanzan a asimilar una cantidad de biomasa necesaria de las presas que permita sostener su población en el tiempo. En la Figura 1 se aprecia como las densidades de predadores van disminuyendo paulatinamente hasta alcanzar la extinción. Dicha situación se repite cuando la capacidad de carga va aumentando en la Figura 4.

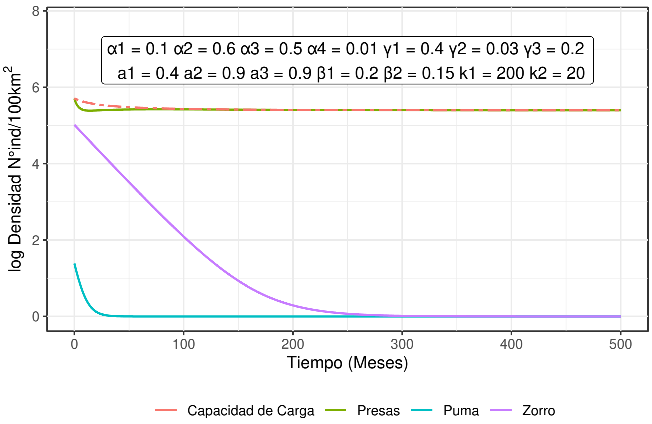

Figura 1: Simulación extinción predadores con K decreciente

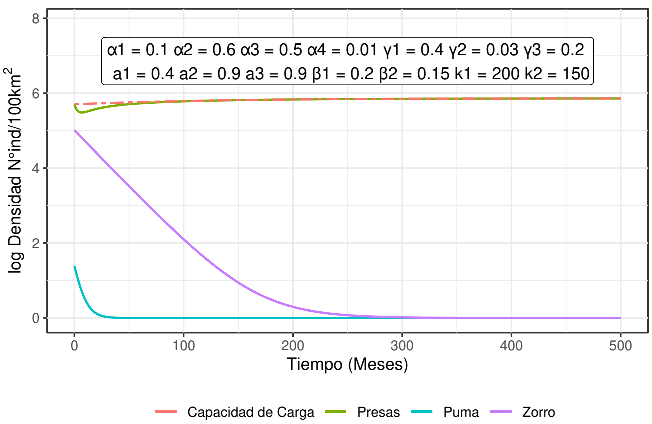

Figura 2: Simulación extinción predadores con K creciente

Las trayectorias de las Figuras 3 y 4 muestran la variación en el tiempo de las densidades de predadores y presas.

Extinción Puma

Las soluciones encontradas surgieron a partir de la asociación de una alta tasa de consumo a Z. culpeo ($\gamma_3 = 0{,}8$), lo cual posibilitó el mantenimiento de la población de zorros

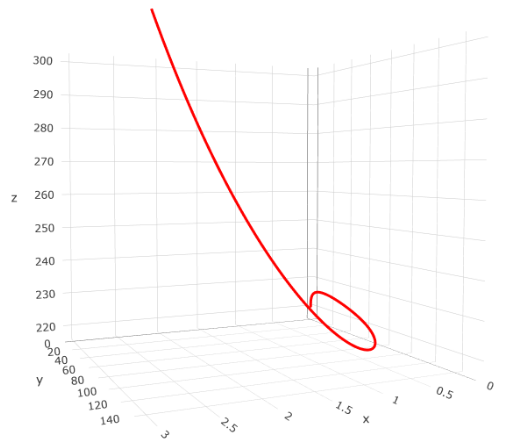

Figura 3: Trayectoria extinción predadores con K decreciente

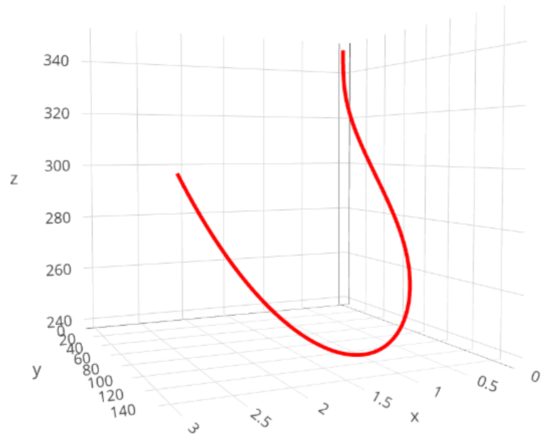

Figura 4: Trayectoria extinción predadores con K creciente

en el tiempo. Como consecuencia, la población de pumas no pudo subsistir. La interacción entre las densidades de Z. culpeo y las presas se muestra en la Figura 5, y se observa también cuando la capacidad de carga aumenta (Figura 6).

Comparando las trayectorias de las Figuras 7 y 8 se aprecia como al estar la capacidad de carga en alza las presas son capaces de tener mayores densidades.

Extinción Zorro culpeo

La extinción de *L. culpaeus*, se asocia a su baja tasa de consumo y a la proliferación de los Pumas. Bajo estas condiciones, se puede apreciar que se genera una retroalimentación entre las densidades de *P. concolor* y sus presas, independiente de la tendencia de la capacidad de carga. Este fenómeno se ilustra en las Figuras 9 y 10.

El cambio de tendencia en la capacidad de carga influye en las densidades máximas de presas que se pueden lograr, pasando de 250 individuos por km² a 350 individuos por km² (Figuras 11 y 12)

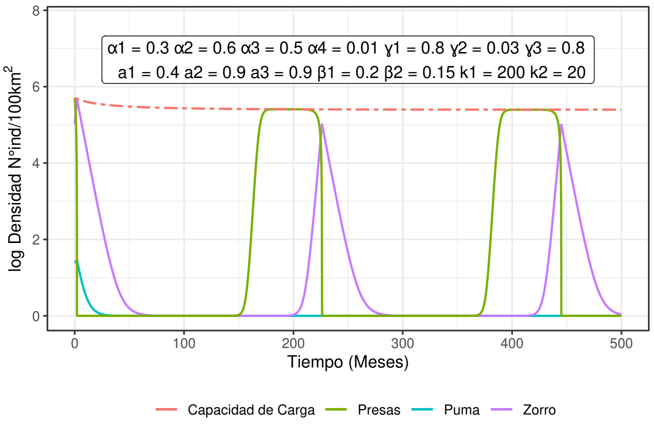

Figura 5: Simulación extinción Puma con K decreciente

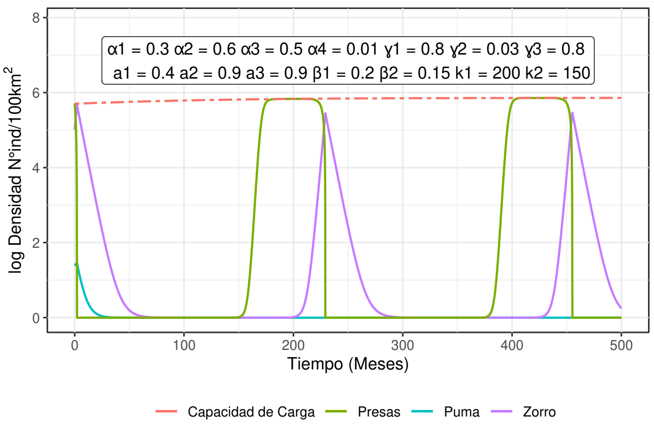

Figura 6: Simulación extinción Puma con K creciente

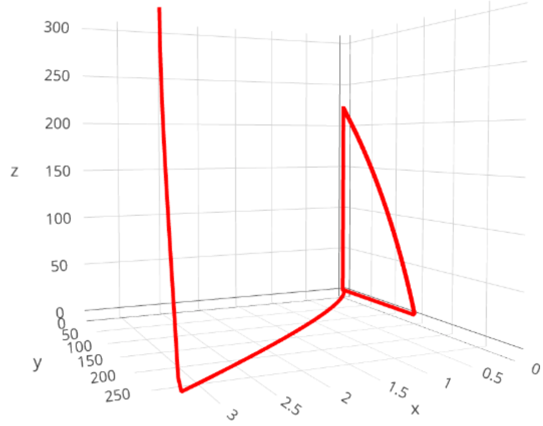

Figura 7: Trayectoria extinción Puma con K decreciente

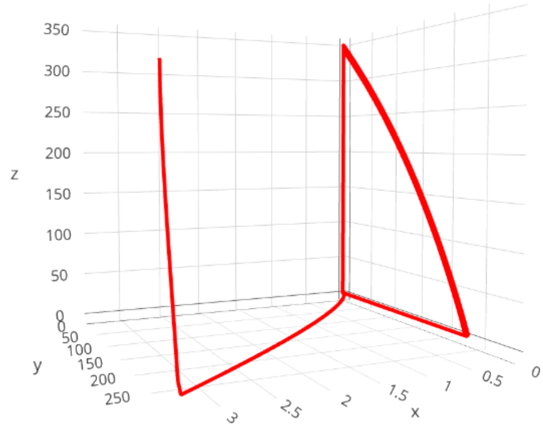

Figura 8: Trayectoria extinción Puma con K creciente

CONCLUSIONES

El comportamiento del sistema ecológico estudiado es altamente dinámico y depende en gran medida de los valores de los parámetros involucrados, especialmente de las tasas de consumo de los depredadores y de la capacidad de carga variable de las presas. Se observa que el aumento en la capacidad de carga puede permitir la recuperación de las poblaciones de presas y, por ende, de los depredadores, siempre y cuando se mantenga un equilibrio adecuado entre ellos. Además, se evidencia que la desaparición de un depredador puede tener un efecto dominante sobre el otro y sobre el ecosistema en general.

Los resultados de este estudio son consistentes con los obtenidos por Al-Moqbali et al. (2018), donde la capacidad de carga variable genera cambios en las oscilaciones de las densidades de predadores y presas antes de que el sistema llegue al punto de equilibrio. Así mismo, fue posible validar la mayoría de los criterios de credibilidad de modelos predador-presa definidos en Berryman et al. (1995).

Es importante señalar que los resultados de este estudio reflejan el principio de exclusión competitiva, lo que significa que no fue posible detectar una solución en donde ambos predadores del sistema coexistieran sin excluirse. Aunque estos resultados están en línea con la ecología de poblaciones, también indican la necesidad de ajustar (2) para permitir estados de coexistencia entre las especies predadoras.

El estudio presentado proporciona información sobre la dinámica de poblaciones en un ecosistema complejo. Aunque es importante destacar que las simulaciones son un modelo simplificado de la realidad y que no se consideran todas las variables que podrían influir en el comportamiento del sistema. Por lo tanto, los resultados obtenidos deben ser interpretados con precaución. Sin embargo, sirven como una base sólida para futuras investigaciones y para la toma de decisiones en la conservación de especies y hábitats.

Es relevante considerar que agregar nuevas variables al modelo y tener en cuenta la tasa de deterioro de la capacidad de carga debido a cambios en el uso de suelos, puede ser un trabajo futuro interesante y pertinente para aplicar la metodología utilizada en este estudio. Con esto, se podrían obtener resultados más precisos que ayuden a mejorar la comprensión y manejo de los ecosistemas en cuestión.

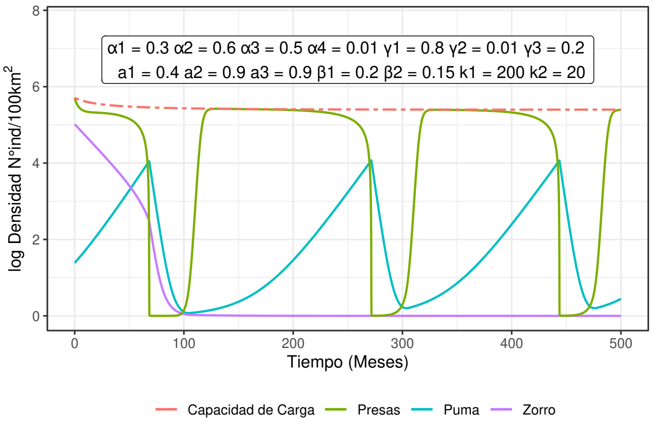

Figura 9: Simulación extinción Z. culpeo con K decreciente

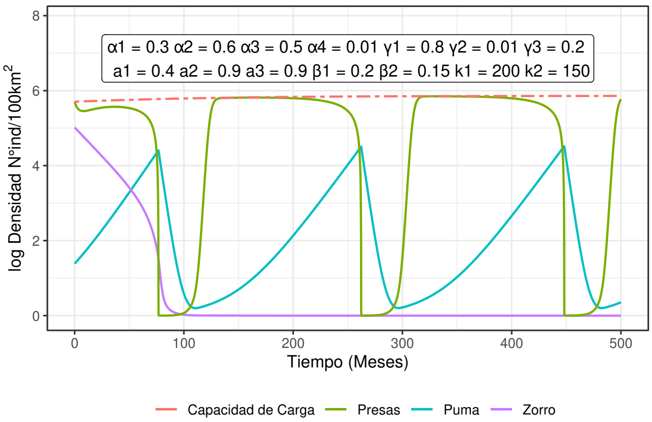

Figura 10: Simulación extinción Z. culpeo con K creciente

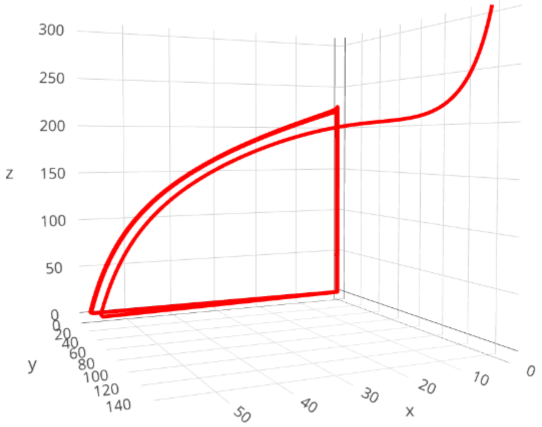

Figura 11: Trayectoria extinción Z. culpeo con K decreciente

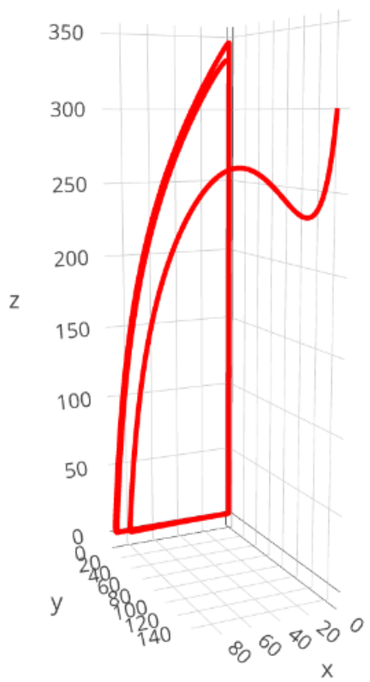

Figura 12: Trayectoria extinción Z. culpeo con K creciente

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