Editorial

MATHEMATICAL MODELS, WHAT FOR?

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Making mathematical models of biological processes is an old scientific activity with a long tradition. In fact, mathematical modeling is a common activity in all sciences, from the "canonical" natural sciences, such as Physics, to the social sciences and humanities. Moreover, we could think that model biological systems can already be considered as a discipline itself, and even an independent science (it has objects of study, methodologies, and a minimal well-defined vocabulary).

But of course, in all scientific activity, it is healthy to become aware and think systematically (and regularly) about the meaning of our task, its why and what for. In this case, we intend to explore some methodological and epistemological issues that seem to still be in discussion or, at least, present diverse interpretations.

Let's see, this is an editorial text that is published in a Journal of Mathematical Modeling of Biological Systems. So, does it make sense to ask about the objectives of modeling? what do we want to achieve when we model mathematically? Why are we particularly interested in modeling these systems and what new, special or important can models contribute in this field?

If we take a brief tour of some fundamental bibliography on the subject, we will see that the interpretations and answers are not uniform and have subtly varied approaches.

For example, Teresa González Manteiga, in her excellent book, places mathematics in a privileged place as a central science. Supported by abundant citations, which include physicists like Galileo or Newton, philosophers like Kant or Manero, and even poets like Paul Valéry, this author places mathematical modeling in the place of discovery and abstraction of the ultimate principles of nature. As we can see, it is a position that goes far beyond the usual instrumental idea of modeling. At the other extreme, we have definitions that point to mathematics as a tool. Hastings, in his book on population dynamics, says succinctly that the goal of population biology is to understand and predict population dynamics and that understanding, explaining, and predicting these dynamics requires mathematical models. As we can see, what is underlined here is an auxiliary role in which the emphasis is placed on the biological problem itself and mathematics takes a subsidiary but useful role because it allows a formalism that facilitates understanding and also quantitative prediction. This approach has been dominant especially in mathematical modeling within population ecology, forgetting in fact that many of the "purely" ecological concepts that are taught as laws have their origin in model analysis, for example, the principle of competitive exclusion in Gause's formulation.

Some other authors are located in an intermediate zone; For example, Gillman and Hails argue that an ecological model must be able to describe the changes in the variables of interest (for example, population density) with some degree of accuracy, and also that such models must be expressed mathematically due to the brevity and formality of the description, the possibility of manipulation of the model, and the possibility of discovering emergent properties that are not apparent to non-mathematical reasoning. As we can see, a fundamental novelty appears here that has to do with the illuminating nature of mathematical models as they are capable of showing what is not seen. This idea underlies other texts, such as the wonderful book by Hernández and Velasco Hernández in which the authors warn us of the dangers of limiting the use of mathematical models to mere prediction and advocate, on the contrary, for more ambitious management where the use of models can even test hypotheses or deepen our understanding of very complex systems. In fact, they present a very nice analogy according to which mathematical models can also be considered an observation instrument: to observe the very distant we can use a telescope; to observe the very small, a microscope; to observe the very complex, mathematical models. It is a provocative and stimulating idea.

In his classic work on models in ecology, Pielou classifies them according to their use. The verbs and expressions used to designate the functions of the models are explain, predict, generate testable hypotheses, serve as ideal patterns against which to contrast real processes. We have here some additional functions to those that we have been commenting on. Can we ask the models for more things? Well, it seems so, because when we open the spectrum beyond the ecological and we take them to all the phenomena that biology studies, we find other topics that must be taken into account to better define the scope of mathematical modeling. For example, in the book by Esteva and Falconi it is stated that "Mathematical modeling offers a research tool that allows the biologist to study the essence of a phenomenon and leave aside details that are not relevant to its understanding", a vision that is closely related to the metaphor that we already mentioned of the observation instrument and with the spirit shown in Manteiga's book advocating mathematics as the privileged path for abstraction and the elaboration of general principles from particular cases. Another very delicate and relevant point that Esteva and Falconi's book raises in its introduction is the deeply interdisciplinary nature of mathematical biology which stresses that "...without a deep knowledge of biology it is impossible to establish a mathematical model and know if it is interesting or irrelevant...".

This brings us to the last two topics that I would like to raise before attempting a synthesis of the matter. First, the problem of the particular nature of biological systems and how this nature influences what we can or cannot do from mathematical models; this is brilliantly developed in an article by Germinal Cocho Gil, published as a chapter of a book coordinated by Sánchez Garduño, Miramontes and Gutiérrez Sánchez. There, the author presents us with two exercises in diatopical hermeneutics posing the oppositions between two schools of evolutionary biology and also the classic Evo-Devo contradiction. Far from exhausting himself in a simple historical description, Cocho shows us issues that are anchored in some essential characteristics of the biological world, in particular the fact that biological systems are historical, dynamic and mutable, and therefore, difficult to classify with static definitions; on the other, the existence of a hierarchy of levels of complexity (which also implies a hierarchy of controls and feedback) in any functioning biological structure. Here the thermodynamic question (once again the interdisciplinary) comes to play an important role and the author associates these questions with what happens at the level of the epistemological discussion of the discipline itself.

The other point that is key to what we want to raise is brought to the fore in Torres Curth's book and is extremely disturbing: the problem of the "truth" of the models. There the author draws our attention to the different qualities of the "truths" of the factual sciences, which depend on facts but are necessarily provisional and have an inductive process behind them, with hypotheses that are put to the test and admit to being refuted; and the truths of the formal sciences that are absolute (either axioms or theorems); that is to say, either because they are accepted as truths for the formal system of which they are part, or because they have been demonstrated from those, the "truths" of the axiomatic systems are forever and do not admit refutation. Can we then adequately represent systems and problems of the natural sciences (which are factual) by means of the objects and laws of the formal sciences (such as mathematics) and achieve a good representation? Some aspects of this problem and its possible answers have been explored by the distinguished colleague and friend Fernando Córdova-Lepe in this same section in the previous issue. There, our colleague emphasizes the problems involved in the interdisciplinary work that is proper to the mathematical modeler who applies his knowledge to biological systems.

TRYING TO UNTANGLE THE THREAD

Having made this necessarily brief review, we can see that the place of mathematical modeling in the biological sciences is manifold and that the themes for meditation that accompany the task are many.

Does mathematics act as a tool when we model biological systems? We could say yes, but it is not always the same type of tool; It depends on the goal we have. It is clear that this tool does not end with the search for predictions only; that the predictions are not only trends or values that could fit better or worse to the empirical data; that the development of mechanistic models of biological processes not only implies a deep understanding of the processes; but also helps that understanding, illuminates aspects, suggests simplifications and generalizations that were not taken into account.

But there is even more, and this is not only valid for the biological sciences but can be thought of in a similar way for other sciences: in a way, the ability to model the biological mathematically opens a door to other possible biological worlds. Somehow, mathematical modeling need not be ulterior to observable biological phenomena; could ask questions about phenomena not yet observed; For example, why aren't there organisms that obtain biological energy by rolling downhill and transforming kinetic energy into chemical energy? And if they did exist, how could they function biologically; What mechanisms should they have, how would they reproduce, what organelles would their cells have, and how would natural selection affect them? Modeling also allows us to imagine organisms and ecologies that could exist in environments on other planets (Carl Sagan pioneered this kind of hypothesis).

On the other hand, it is quite common that the best-known biological problems, when trying to be modeled, also pose particular mathematical problems that sometimes lead us to the development of techniques or to the rescue of somewhat forgotten areas of mathematics. A possible example of topics that are requiring the development of new mathematical techniques or, even, the development of new concepts, is the broad development that the study of biological networks is undergoing, from interaction networks between species (trophic webs, competition networks , mutualistic networks) to metabolic networks, genetic regulation, social relations between animals, etc. One of the difficult issues to resolve in the study of these dynamic structures is to determine their stability in the face of external disturbances; another is the field of predicting the dynamics of these networks. Currently, the mathematical tools we have have proven to be insufficient or not very subtle to capture these complex dynamics. Stability metrics are multiplied and are supported by a multiplicity of auxiliary hypotheses that are not always plausible, or are obtained from reductionist simulations (which underestimate non-linear interactions). Surely there are properties of matrices that can be associated with variables analogous to free energyi and that would be worth exploring in multidisciplinary teams.

In summary: our area of work, which has an academic guideline in this journal, is increasingly broad, challenging and provocative. And a fertile territory for another of the human capacities that is at the base of all science: imagination.

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Notas al final

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